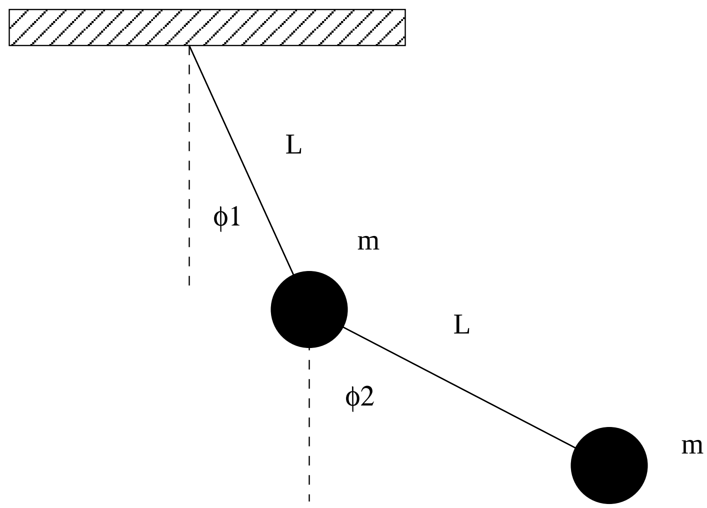
Physical Mechanics II 33-332, Recitation 14 (Wed May 3 2006)

This is a recitation question and not a homework question. You will be asked to work in groups to solve this question on the day of the recitation session. The question will be graded. The grade will be out of 5, 4 points for handing something in, and 1 for your answer to the problem itself). Please feel free to work on the question in advance.

(1) Consider the case of the double pendulum shown below. The motion is only in the plane, and the masses and bob lengths are equal.

(a) Assuming small oscillations, find the normal modes and coordinates.



(b) For this part of the question, just read the following, and then either try out the demo pendulum in class, or else this web applet:

http://www.ifm.ethz.ch/meca/applets/doppelpendel/dPendulum.html

If the amplitude of oscillations is not small, then the equations of motion for $\phi_1(t)$ and ϕ_2 must be solved numerically. The equations are non-linear (i.e. $\dot{phi}_1 = f(\phi_1, \phi_2)$, where f is not a linear function of ϕ_1, ϕ_2), and in this case the motion is not periodic, but appears random or erratic. The system exhibits chaotic behaviour, which can be characterized by its sensitivity to initial conditions:

To identify chaos, take two initial conditions differing by a small amount:

$\phi_{1,0}$ and $\phi'_{1,0} = \phi_{1,0} + \epsilon$.

and compare the differences $d_n = \phi'_{1,0} - \phi_{1,0}$ with exponential growth.

For exponential growth you should see $d_n \simeq \epsilon e^{n\lambda}$, where λ is the average rate of the exponential growth, known as the Lyapunov exponent. If λ is positive, the paths are divergent, if negative, the system is non chaotic.