

# THE ZEEMAN EFFECT

V2.0

R. A. Schumacher, April 2008

B. B. Luukkala, January 2001

## I. INTRODUCTION

The goal of this experiment is to measure the Bohr magneton using the *normal Zeeman effect* of the 643.8 nm (red) line of cadmium and the *anomalous Zeeman effect* of the 508.6 nm (blue) line of cadmium. Zeeman splitting of these spectral lines is observed using a piezo-electrically tuned Fabry-Perot interferometer.

## II. BACKGROUND READING

The following references may provide useful background for the experiment and should be perused:

| <u>Reference</u>                 | <u>Sections</u> | <u>Pages</u> | <u>Topics</u>              |
|----------------------------------|-----------------|--------------|----------------------------|
| Required:                        |                 |              |                            |
| Melissinos <sup>1</sup>          | 1.6             | 33 – 43      | Atomic spectra             |
|                                  | 6.2 – 6.5       | 218–243      | Zeeman experiment          |
|                                  | 4.6             | 172–177      | Fabry-Perot Interferometer |
| Recommended:                     |                 |              |                            |
| Park <sup>2</sup>                | 7.7             | 251–255      | Zeeman effect              |
|                                  | 10.4            | 388–394      | Selection rules            |
| Feynman, vol. III <sup>3</sup>   | 12-4            | 12–9,12      | Zeeman theory              |
| Eisberg and Resnick <sup>4</sup> | 10-6,7          | 364–371      | Zeeman theory              |

The basics of the level-structure of cadmium can be (re)learned from Section 1.6 in Melissinos. There, the major levels and fine structure splitting of *mercury* are discussed, but mercury and cadmium both have effectively two active outermost electrons, making their structure very similar to each other and to that of helium. This means that their spectroscopies are very similar. This section also discusses the selection rules for transitions among atomic energy levels.

Almost any quantum mechanics textbook will discuss the Zeeman effect at some level. The introduction in Melissinos is fine, but you may want to start with your own quantum physics textbook.

### III. THEORY OVERVIEW

The Zeeman effect is the breaking of degeneracy in atomic energy levels due to the interaction between the magnetic moment of an atom and an external magnetic field. The strength of this interaction in each energy state depends upon the total angular momentum of the atom, given by

$$\vec{J} = \vec{L} + \vec{S} \quad (1)$$

where  $\vec{L}$  and  $\vec{S}$  are the orbital and spin angular momenta, respectively, and where  $|\vec{J}|$  can take on values from  $|\vec{L}| - |\vec{S}|$  to  $|\vec{L}| + |\vec{S}|$ . The z-component of  $\vec{J}$  is labeled by the quantum number  $m_J$ , and is restricted to discrete values, in integer increments, in the range given by

$$|\vec{J}| \leq m_J \leq |\vec{J}| \quad (2)$$

Thus, there are  $2J+1$  allowed orientations of  $\vec{J}$ . In zero external field there is no energetically preferred orientation of  $\vec{J}$ , and the energy states of the atom are said to be degenerate in  $m_J$ , which means they are all the same.

The presence of an external magnetic field defines a preferred axis, namely the direction of the field. Each allowed orientation of  $\vec{J}$  with respect to this axis is associated with a different energy, thus breaking the degeneracy. An easily observable result of this is the splitting of a spectral line into several component lines, the number of which depends upon the spin and orbital angular momenta in the initial and final energy states involved in producing the spectral line.

The so-called *normal* Zeeman effect refers to the splitting of a spectral line into three components - a phenomenon which was explained classically in terms of changes in frequency of orbit of an electron in the presence of a magnetic field. This effect is observed in spectral lines for which the initial and final energy states have zero spin angular momentum.

By contrast, the anomalous Zeeman effect refers to the splitting of spectral lines into more than three components. The term 'anomalous' was given because the phenomenon, which had been observed since the late 1890s, was unexplained until the discovery of electron spin some thirty years later.

The energy of interaction between a magnetic moment  $\vec{\mu}$  and an applied magnetic field  $\vec{H}$  is given by

$$E_{mag} = \vec{\mu} \cdot \vec{H} \quad (3)$$

As customary, we choose the z-axis of our coordinate system to be along the direction of  $\vec{H}$  which then gives

$$E_{mag} = \mu_z H \quad (4)$$

or in terms of the z-component of the angular momentum

$$E_{mag} = g \mu_0 m_J H \quad (5)$$

where  $g$  is the ratio of the magnetic moment of the whole atom to the fundamental constant

called the Bohr magneton,  $\mu_0$ . (The minus sign has disappeared since for electrons the direction of the magnetic moment is opposite to the direction of the angular momentum.)

**Exercise 1:** Show from first principles that the “natural” unit of magnetic moment of a classical unit electric charge of mass  $m$  orbiting a “Bohr” atom is given by

$$\mu = \frac{e \hbar}{2m} \quad (6)$$

The g-factor for a free atom in a given state  $^{2S+1}L_J$  can be calculated according to

$$g = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)} \quad (7)$$

(as derived in, e.g. Eisberg & Resnick, p.368, or Melissinos p.225.) Herein is the key to understanding the difference between the normal and the anomalous Zeeman effects.

As mentioned earlier, spectral emission lines which exhibit the normal Zeeman effect involve transitions between energy states which have zero spin angular momentum,  $\mathbf{S}$ . Thus, the orbital angular momentum,  $\mathbf{L}$ , and the total angular momentum are identical, and we have  $g=1$  for both energy states. The energy shift for a given value of  $m_J$ , given by Equation (5), is the same in both the upper and the lower energy state. In the electric dipole transition process (which is the dominant process for this source) the change in  $m_J$  is restricted according to the selection rule

$$\Delta m_J = +1, 0, -1 \quad (8)$$

As a result, when a magnetic field is applied the zero field line is observed to split into only three separate components (each of which may still have some degeneracy!).

Spectral lines which exhibit the *anomalous* Zeeman effect involve energy states with nonzero spin angular momentum,  $\mathbf{S}$ . Electric dipole transitions do not involve a change in spin, but do involve a change in orbital angular momentum according to the selection rule

$$\Delta L = \pm 1 \quad (9)$$

Because of this, the g-factors will be different for the upper and lower energy states, and the resulting splitting of the zero field line in the presence of a magnetic field may be more complex.

Both the *normal* and the *anomalous* Zeeman effects are observable in the visible portion of the spectrum of cadmium. The normal effect is exhibited by the 643.8 nm (red) line produced by transitions from the  $5^1D_2$  state to the  $5^1P_1$  state. The anomalous effect is exhibited by the 508.6 nm (blue) line produced by transitions from the  $6^3S_1$  state to the  $5^3P_2$  state. Energy level diagrams for these transitions are shown in Figures 1 and 2, below.

**Exercise 2:** Review the meaning of the  $^{2S+1}L_J$  spectroscopic notation used in the discussion above and below. Then explain to your instructor whether or not  $^3S_0$  or  $^1D_2$  states are possible.

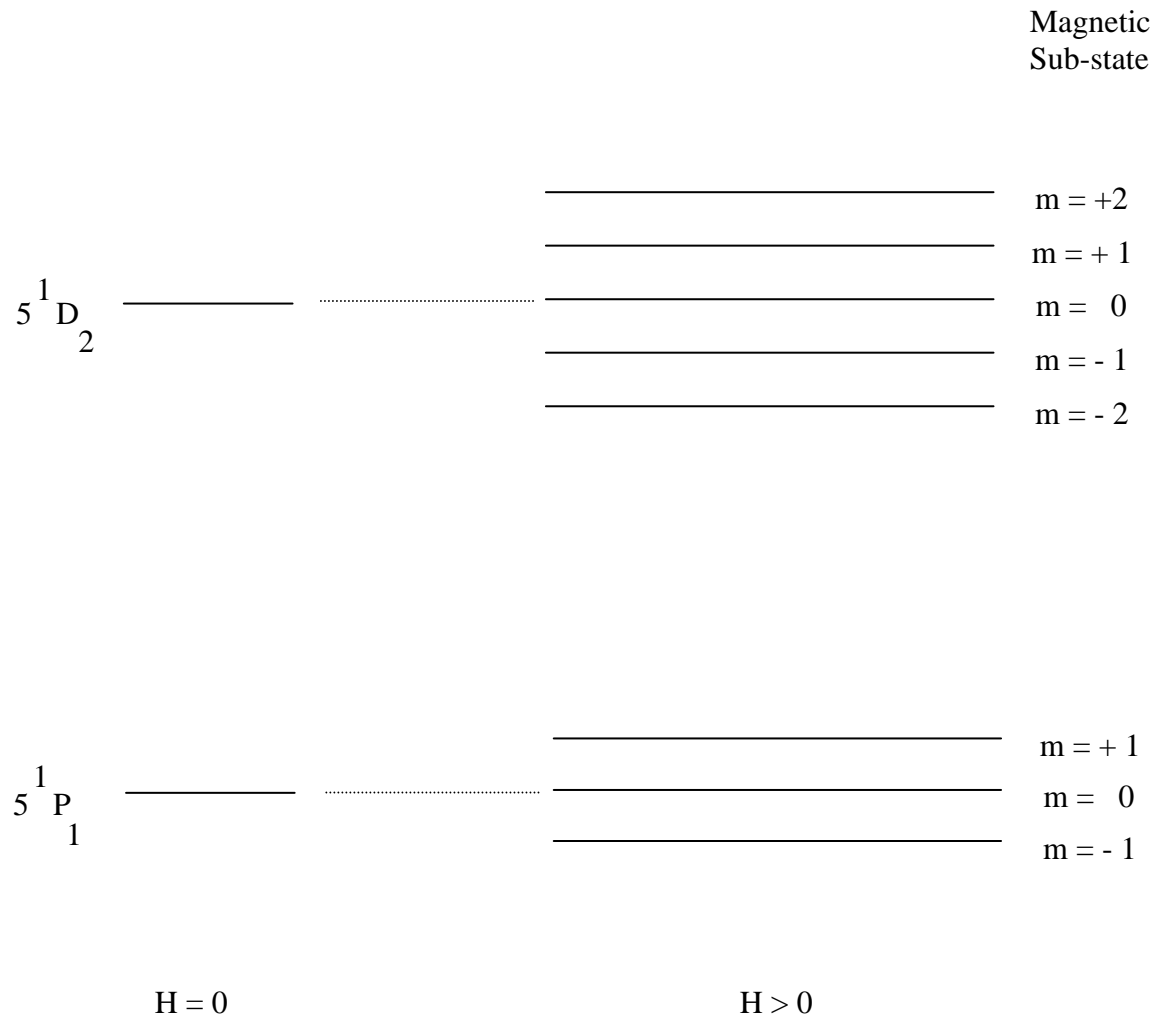


Figure 1.

The normal Zeeman effect is exhibited by the 643.8 nm line of cadmium, since the spin angular momentum is zero for both the upper and lower states. There are nine allowed transitions: three each for  $\Delta m = 0, +1,$  and  $-1,$  but with a three-fold degeneracy for each value of  $\Delta m.$  As a result, the zero field spectral line is observed to split into only three components.

**Exercise 3:** Calculate the  $g$ -factors for the two zero field energy levels in Figure 1 and draw vertical lines indicating the allowed transitions between the various Zeeman-split energy levels. Convince yourself that there are nine allowed transitions, but that the three-fold degeneracy results in only three unique emission lines in nonzero field.

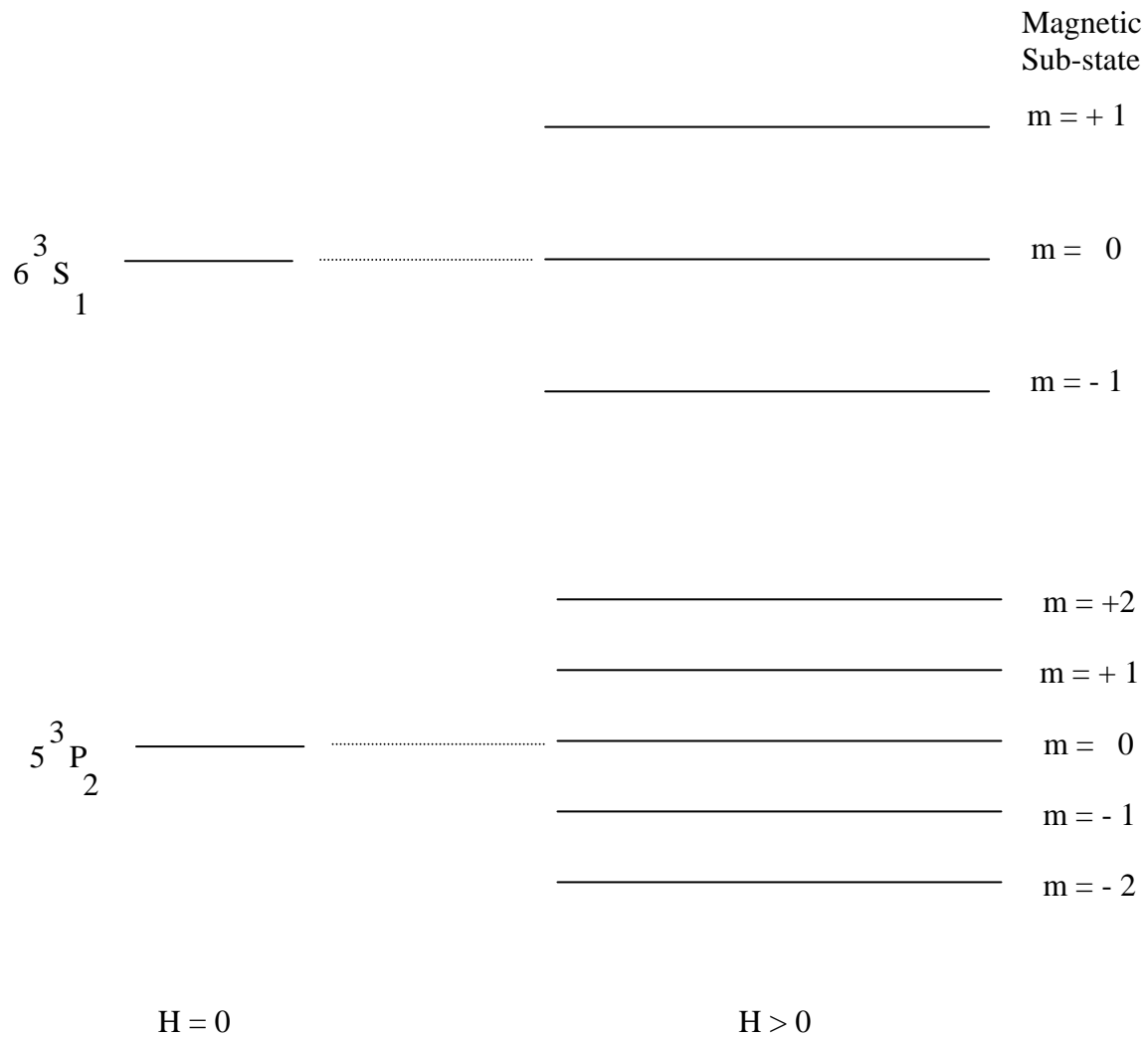


Figure 2.

The anomalous Zeeman effect is exhibited by the 508.6 nm line of cadmium. The spin is nonzero and the initial and final  $g$ -factors are different. In the presence of a magnetic field the nine-fold degeneracy is completely broken and the zero field line is observed to split into nine components.

**Exercise 4:** Calculate the  $g$ -factors and draw in the nine allowed transitions as you did for the example in Figure 1.

## IV. EXPERIMENTAL APPARATUS

The experimental setup is very similar to that in Melissinos, and is shown schematically in Figure 3. The significant difference in our setup is in the imaging system used for viewing and recording the interference patterns. We use a CCD video camera, monitor and video printer in place of the projection lenses, prism spectrograph and photographic plate used in Melissinos. This makes the alignment of the optics much easier and eliminates the need for photographic processing.

In addition, there is an electromagnet for producing a strong field over the region of space that the light is emitted. The magnet is powered by a supply calibrated in Amperes of current. You will have to determine the actual magnetic field strength using a Hall probe Gaussmeter.

The Fabry-Perot interferometer is the heart of the apparatus which allows us to detect small shifts in the wavelengths of light emitted from the source. Basically, it is a pair of parallel highly reflective mirrors separated by distance  $t$ . Study the relevant material in Melissinos. Using what you learn, answer the following questions:

### **Exercise 5:**

- (a) Given a single wavelength of light entering the Fabry-Perot interferometer, will there be a single ring, or a whole family of rings?
- (b) If all the light entered the interferometer at exactly the same polar angle,  $\theta$ , and at only a single azimuthal angle,  $\phi$ , what would the pattern of light on the screen look like?
- (c) In terms of  $\lambda$ ,  $t$ , and  $\theta$ , what is the angular distance,  $\Delta\theta$ , between rings in the pattern?
- (d) If the wavelength of light were to shift by a small amount  $\delta\lambda$ , by how much an angle,  $\delta\theta$ , will the ring pattern shift?
- (e) If the wavelength,  $\lambda$ , were to shift enough,  $\Delta\lambda$ , to move a ring to the initial position of a neighboring ring, show that  $1/2t = \Delta\lambda/\lambda^2$ .

In what follows below, we will need the ratio of  $\delta\theta$  to  $\Delta\theta$ , which we will call  $\alpha$ . For this experiment, it is the ratio of how far Zeeman splitting shifts a ring in units of the natural ring spacing between rings in the interferometer. You can also show that in terms of frequency differences,  $\Delta\nu$ , rather than wavelength differences,  $\Delta\lambda$ , the distance between rings is  $c/(2t)$ . The jargon name for this frequency difference between rings is called the “free spectral range” of the interferometer, and the larger it is, the more sensitive the interferometer is to small changes in frequency.

## V. EXPERIMENTAL PROCEDURE

**The following general results are to be obtained for this experiment:**

- 1) Get a value for the Bohr magneton for the normal Zeeman effect using one spacing of the Fabry-Perot interferometer.
- 2) Get another measurement of the Bohr magneton for the anomalous Zeeman effect using one spacing of the Fabry-Perot interferometer.
- 3) Repeat your measurements for other plate spacings.
- 4) Compare your results to seek possible sources of systematic errors.

**More detailed steps to get the results:**

1) Calibrate the magnetic field. Determine the relationship between power supply current and magnetic field using the Hall probe. Be very careful with the Hall probe because it is delicate: don't bend it.

2) Aligning the optical train. First, check to be sure that all of the optical components are properly aligned along a common axis. To do this, just turn on the cadmium lamp power switch and allow the lamp to warm up (about 5 minutes should be enough). Then turn out the room lights and probe the beam path with a small piece of white paper to be sure that the light is passing through the center of each optical component.

3) Video Camera Lens. Plug in and turn on the video camera and monitor. Then familiarize yourself with the video camera lens adjustments:

- The focus ring is the one farthest from the camera body. The markings on this ring indicate a range of focus from slightly less than 1 meter to infinity. Focus the lens near infinity during data taking.
- The Focal Length ring ("Zoom") is the middle ring with markings from 12.5 to 75 (mm). Start with wide-angle focal length (somewhere between 12.5 and 20 mm).
- The ring closest to the camera body is the f-stop, which controls the size of the aperture, and hence the amount of light entering the camera. The f-stop is the focal length divided by the diameter of the clear aperture. So a smaller f-stop allows more light into the camera. Adjust this at any time as you see fit.

4) Aligning the Fabry-Perot Plates. The plates of the Fabry-Perot etalon must be made parallel with great care in order to achieve the resolution necessary to observe the Zeeman effect. This is done in three stages:

- a. Coarse adjustment with micrometer screws; camera on wide-angle view.
- b. Finer adjustment with micrometer screws; use a longer focal length.
- c. Fine-tuning with piezoelectric tuner; camera at longest focal length.

a. Coarse adjustment: First, turn on the piezoelectric tuner power supply and set all three knobs at mid-range. Pull back lens L2 until you see an image of the lamp on the monitor. Adjust the focus the video camera as needed to see sharp edges. Use the micrometer screws at the back of the Fabry-Perot to overlap the multiple images of the source. One of the

micrometer screws will spread or compress the spot horizontally, while the other screw will do the same vertically. A sign that you have it pretty well adjusted will be the sudden appearance of a ring pattern on the monitor. At this point, move lens L2 back within about 6 cm of the Fabry-Perot.

b. **Finer adjustment:** Put the camera focus near infinity. Use the camera focal length ring and zoom in to about 40mm. Continue to adjust the micrometer screws (very sensitive at this point) to get the ring pattern to be as uniformly bright and sharp as possible.

c. **Fine tuning:** Zoom all the way to 75mm on the camera focal length. Use the x- and y-knobs on the piezoelectric tuner to further sharpen up the ring pattern. Camera focusing will help. A successful alignment has a pattern of very sharp and narrow rings which are uniformly bright around their entire circumference.

## VI. MEASUREMENTS AND ANALYSIS

We have not given you the full working equation for determining the Bohr magneton. This is up to you to figure out from the information given above and in the references. It depends on the g-factors of the states involved, the field strength, and the amount of fringe shift. You will quickly learn that the key idea is to determine how far a "ring" in the interference pattern moves for a given change in field strength. The idea is to shift the ring positions until neighboring orders of rings overlap, creating a brightening of the observed patterns. From the amount of the shift, measured in units of ring spacings,  $\alpha$ , you can deduce the Bohr magneton. In practice, you will let the rings shift by large amounts: half, whole, or  $3/2$  free spectral range shifts are typical. This is because such big shifts are easy to identify on the screen.

***Exercise 6:** Make a graph in your log book, using pencil and ruler, with ring location on the vertical axis and magnetic field strength on the horizontal axis. For any given transition you are studying ("red" or "blue", polarized one way or the other) predict how the lines will split as a function of  $H$ . This will help you interpret what you see on the screen.*

Each of the rings in the Fabry-Perot interference pattern is multiply degenerate when the external magnetic field is zero. When a magnetic field is applied to the cadmium atoms in the gas discharge tube the degeneracy is broken, and each of the rings in the interference pattern will split into several rings, depending on which filter and polarization you have selected. As you increase the strength of the magnetic field the new rings will move away from the zero field rings in both directions (to higher and lower wavelength). The energy difference between the zero field transition and any of these shifting rings will be the sum of two terms, each given by Equation (5). One term comes from the energy shift of the upper state, and the other from the energy shift of the lower state. The idea is to cause the moving rings to shift into an easily recognizable pattern of overlap, so that you know exactly how much shift in wavenumber has occurred. For example, apply enough magnetic field strength to shift the outermost ring in each order of interference by exactly  $1/2$  the free spectral range, and it will overlap with the innermost ring from the next order of interference exactly half way in between the original ring positions.

You can relate the observed shift in wavenumber to the Zeeman shift of energy states. In general, you will have expressions of the form

$$\beta \frac{\mu_0 H}{h c} = \alpha \frac{1}{2t} \quad (10)$$

where  $\beta$  involves the g-factors and z-components of angular momentum for the upper and lower energy states of the transitions you are observing, and  $\alpha$  will usually be 1/2 or 1 or 3/2, depending on the pattern of overlap which you have created (*i.e.* 1/2 for shifts which produce overlaps half way in between the zero field rings, and 1 for overlaps at the original zero field positions, etc.). Thus, for any matching combination of  $\beta H/hc$  and  $\alpha/2t$  you can compute a value of  $\mu_0$ .

The recommended procedure is to choose a plate separation for the Fabry-Perot (anything between 0.5 and 1.0 cm is good to start), align the plates and measure the separation,  $t$ . Then observe as many patterns of overlap as you can achieve for both the *normal Zeeman effect* (red filter) and the *anomalous Zeeman effect* (blue filter) for both orientations of the polarizer ( $0^\circ$  and  $90^\circ$ , corresponding to the  $\pi$  and  $\sigma$  transitions, respectively). Record the magnet current for each pattern of overlap. Devise some procedure for measuring the uncertainty in the current reading. In other words, how reproducible is a given pattern of overlap, in terms of the field strength needed to create it?

**Exercise 7:** For each  $\Delta m$  value, what are the relative orientations of (a) the  $H$  field, (b) the light propagation direction, (c) the electric field vector of the light? You should get the answer to this question from your readings on Zeeman Effect theory.

After you have obtained all of the data possible for one plate separation, then move the plates to a different separation to obtain more measurements. (Remember to remeasure the plate separation after each alignment.)

At some point you will need to calibrate the magnet by plotting the field strength, as measured with the Gaussmeter, as a function of the magnet current. It is up to you whether you want to do this first, or just record the current used for each pattern of overlap, and later relate that current to a magnetic field strength.

Report your final results as a measurement of the Bohr magneton,  $\mu_0 = e\hbar/2m_e$ , or alternatively as a measurement of  $e/m$ . Compare your results to previously measured values, which may be found on page 243 of Melissinos. Critically evaluate your results in terms of their internal consistency (repeatability), other systematic uncertainties, and comparison with the world average values.

## References

- 1) A. C. Melissinos and J. Napolitano, *Experiments in Modern Physics, 2<sup>nd</sup> Ed.* Academic Press, New York, 2003. (The older first edition is more or less the same, though the page number references are all different.)
- 2) David Park, *Introduction to the Quantum Theory*, McGraw Hill, 1974.
- 3) R. Feynman, R. Leighton, M. Sands, *Lectures on Physics, vol. III*, Addison-Wesley, 1965.
- 4) R. Eisberg and R. Resnick, *Quantum Physics, 2nd ed.* Wiley, 1985.
- 5) P. R. Bevington and D. K. Robinson *Data Reduction and Error Analysis for the Physical Sciences*, McGraw Hill Book Company, New York, 1992.

