I. INTRODUCTION

When a substance is placed in an external magnetic field \( \vec{H} \), it becomes magnetized (there is an induced magnetic moment, \( \vec{M} \)) to a varying extent, depending on the substance. This is analogous to the electric polarization of a dielectric when it is placed in an electric field. We define the magnetic susceptibility to \( \vec{H} \) and call it \( \chi \)

\[
\vec{M} = \chi \vec{H}
\]

In general \( \chi \) is a tensor, which is to say that a vector external field \( \vec{H} \) produces a magnetization in the material, \( \vec{M} \), which is also a vector but may not point in the same direction as \( \vec{H} \). For our samples, however, \( \vec{M} \) and \( \vec{H} \) are parallel and \( \chi \) is a simple scalar.

A. Energies of interaction

The magnetic behavior of a given material depends on three different interactions, in competition with one another. While the external magnetic field is trying to align all the magnetic moments in the sample, entropy resists that alignment. This competition is described in more detail in Appendix 1. If thermal energy and the interaction of the spins with the external field are the only two relevant energies of interaction, the material is said to be paramagnetic, and the magnetic susceptibility follows the so-called Curie Law:

\[
\chi = \frac{C}{T}
\]

where \( C \) is the Curie constant and \( T \) is the temperature. To put it simply, the greater the thermal energy, the less susceptible the material is to being magnetized.

However, this classical model does not tell the whole story. There is an additional interaction between neighboring unpaired electron spins within the crystal lattice – the quantum mechanical exchange interaction. At sufficiently low temperature (note this can be room temperature for some materials), this exchange interaction may cause spontaneous alignment of the spins with one another, all in the same direction, with no applied field, \( i.e. \chi \) goes to \( \infty \), or it may cause the spins to anti-align with one another within the crystal lattice, \( i.e. \chi \) tends toward zero. The details of the crystal structure (\( i.e. \) the lattice constants and crystal symmetry) determine which of these two possibilities will result.
The case in which neighboring spins align with one another in the same direction is referred to as ferromagnetism. A phenomenological model, the Curie-Weiss Law, describes the temperature dependence of $\chi$:

$$\chi = \frac{C}{T - T_c}$$

where $T_c$ is the Curie temperature below which $\chi$ appears to go to $\infty$.

If the interparticle interaction results in antialignment of neighboring spins with one another (i.e. the spins alternate in opposite directions throughout the crystal lattice below a certain temperature), the material is said to be antiferromagnetic. This behavior is modeled (above some critical temperature) by the so-called Curie-Neél law:

$$\chi = \frac{C}{T + \theta}$$

where $\theta$ is the Neél temperature, below which $\chi$ tends toward zero.

B. Diamagnetism
Some materials have no unpaired electrons, yet the magnetic susceptibility is, in general, not equal to zero. Orbital angular momentum results in a net magnetization of the sample in a direction opposite to the applied magnetic field (i.e., $\chi < 0$). Such materials are said to be diamagnetic, an effect which may be several orders of magnitude weaker than paramagnetism. A classical analogy to diamagnetism is that of a gyroscope precessing in a gravitational field.

C. Experimental technique
One way to measure the magnetic susceptibility, $\chi$ of a material is the Gouy balance method: a long thin sample is placed in a magnetic field gradient (pointing along the length of the sample) and the force on the sample is measured. The attached article by McMillan explains the method. (You may remember the similar problem of a dielectric slab placed in the fringe field of a parallel plate capacitor.)

D. Questions to be addressed
Read the appendices to this write-up and write brief answers to the following questions in your lab book.

**Exercise 1:** Extend the equation for the effective mass of the sample to include the fact that the sample is sealed in a fused quartz tube that has a magnetic susceptibility of its own.

**Exercise 2:** Consider the Langevin function that describes the magnetic susceptibility for a paramagnet as a function of temperature. (a.) Would you expect a larger or smaller force on the sample at fixed $H$ but lower temperature? (b.) If the Curie law holds, how does $k_BT$ compare to the energy of the magnetic dipole in the external field? In what regime of the Langevin function are your experiments occurring?
**Exercise 3:** What are the differences between paramagnetism and diamagnetism (both in relative magnitude and in sign)? How will you be able to discern these differences experimentally?

**Exercise 4:** What is the difference between ferromagnetism and antiferromagnetism?

**Exercise 5:** From measurements of $\chi$ at two temperatures (room temperature and 77K), how will you solve for $C$ and $T_c$ or $\theta$? Note that both $T_c$ and $\theta$ are positive quantities. How will you be able to tell if the material would become ferromagnetic or antiferromagnetic at low temperatures?

**II. MEASUREMENTS TO BE MADE**

The experiment requires that samples be measured at two temperatures: room temperature and liquid nitrogen temperature. You will examine four materials (plus a blank fused quartz tube). First, by plotting $m$ vs $H$, you will find $\chi$ at each temperature. Then you will use your measurements to determine $C$ and $T_c$ or $\theta$ for each material. The samples which must be measured are:

1. Mn CO$_3$ - manganese carbonate
2. Fe (SO$_4$)$_2$ (NH$_4$)$_2$ • 6H$_2$O - Mohr’s salt
3. NiSO$_4$ • 6H$_2$O - nickel sulfate
4. Zinc
5. Empty fused quartz tube for correction for the susceptibility of the tubes containing the samples.

**III. METHODS**

**A. Experimental Apparatus**

The balance and sample are enclosed in a large lucite box and a glass tube extending into a dewar, which when filled with the liquid nitrogen cools the sample. The balance rests on a plastic platform which has two brass tubes extending below it: one is used to pass gaseous nitrogen and the other positioned below the balance pan connects to the sample tube. The sample tube and the dewar can move forward to allow insertion of samples and give access to the various holes in the dewar. The magnet can be rotated.

**B. Measuring the Magnetic Field**

The gaussmeter probe will be mounted to the side of the dewar during the experiment. But you will need the magnetic field at the top and bottom of the sample when it is hanging from the balance inside the dewar. Moving the dewar out of the way and using the special meter stick provided, measure $H$ at: (i) the pole face where the probe will stay during the measurements ($H$), (ii) the center of the pole gap where the bottom of the sample will hang ($H_B$), and (iii) at the position of the top of each of the four samples ($H_T$). Carefully obtain values and uncertainties for the ratios $H_B/H$ and $H_T/H$ at several magnet currents. (The current is conveniently determined by the voltmeter attached across a shunt resistor in the magnet power supply.) Hopefully, the ratio is independent of the magnet current. **Be sure to turn on the cooling water before turning on the magnet power supply.** Rewrite your equations for the effective mass in terms of these ratios.
C. Measuring the Effective Mass of Each Sample at Room Temperature
Carefully hook the sample on the hook under the balance. **DO NOT APPLY DOWNWARD FORCE ON THE BALANCE AS YOU HANG THE SAMPLE.** Slowly lower the sample into the dewar, push the dewar back, pull the brass collar up, and make sure the sample is hanging freely in the dewar.

Set the magnet current and note the field and force. There are drifts in the apparatus so you need to work with care, repeating some fields and taking measurements at random field values rather than monotonically moving H in time. Using a least squares fit to this data, determine $\chi$ for each material.

D. Measuring the Effective Mass of Each Sample at Liquid Nitrogen Temperature
Liquid nitrogen is used to cool the sample to about 77 K. At this temperature oxygen from the atmosphere condenses into a liquid, and will then condense inside the tube containing the sample unless we eliminate oxygen from the tube. We do exactly that by flushing out the system (balance and sample lube) with helium gas. The important idea to remember is that little oxygen should enter the system after the liquid N$_2$ is in the dewar. A very rapid flow of He gas is NOT required to achieve this result. Think in terms of conserving the gas as well as flushing the system. A schematic diagram of the gas lines is shown in Figure 1.

The following experimental procedure may or may not be followed. It is just a suggestion.

1. Close pinch cock to allow flow of gas through balance cover. Use gentle flow; conserve the gas.

2. Remove wire hook from balance pan.

3. Insert stopper (#4 solid) into brass pipe under balance pan.

4. Open pinch cock and run long length of clear plastic tubing down the sample tube (the long tube which isolates sample from dewar).

5. Gas should be on. Slowly remove the tubing leaving a trail of He gas behind.

6. Quickly place #4 one hole stopper over top of sample tube and connect plastic tubing to it. This keeps the He gas in the system under positive pressure to prevent oxygen from entering.

7. Fill dewar with liquid N$_2$. A quart thermos is provided. Fill dewar with enough liquid N$_2$ to ‘cover’ the sample. Please see instructor for procedure for obtaining the liquid N$_2$.

8. A funnel connected to a black rubber is then placed in the forward most part in the top of the dewar assembly.

9. Have vent tube open. Leave all other stoppers in place. The dewar should now be forward on its track.
10. With funnel held high (level with balance platform) climb on ladder and pour the liquid N\textsubscript{2} slowly into funnel, allowing it to cool and become brittle. Funnel may be clamped to the frame. Do not splash the liquid N\textsubscript{2} into lab partner’s eyes. Carefully fill the dewar to a level just above the top of the sample.

11. After dewar is filled to level desired, use some devious method to get sample into sample tube without letting in much air, for example surround entrance of the tube with flow of He gas.

12. Remove cork from under balance and replace hook. He gas still flowing gently. Seal off system, sliding up brass collar.

13. Make measurements after waiting long enough for the sample to get cold.

14. When changing samples, again have a gentle flow of He gas around the top of the tube to keep air out.

\textit{E. Determine C and T\textsubscript{c} or }\theta\\
From your measurements of \(\chi\) at each temperature, determine values and uncertainties for C and T\textsubscript{c} (or \(\theta\)) for each material. Discuss whether you can tell if each material will become ferro- or antiferromagnetic at low temperatures.
Diagram of the Guoy Balance Apparatus

- Balance Chamber (flush with Helium for low-temp measurements)
- Analytical Balance
- Helium gas cylinder
- Electromagnet (water-cooled)
- Inner Sample Tube (couples to balance chamber)
- Double-walled Dewar (holds Liquid Nitrogen)
- Sample
- Gauss Meter for Field Measurement
APPENDICES:

- J. A. McMillan “Apparatus for the Determination of Magnetic Susceptibilities”
- Units for Magnetic Properties
- Chapters from Charles Kittel, “Introduction to Solid State Physics,” 3rd edition
Equipment for the Determination of Magnetic Susceptibilities

J. A. McMillan

Citation: American Journal of Physics 27, 352 (1959); doi: 10.1119/1.1934850
View online: https://doi.org/10.1119/1.1934850
View Table of Contents: http://aapt.scitation.org/toc/ajp/27/5
Published by the American Association of Physics Teachers

Articles you may be interested in

Determining magnetic susceptibilities of everyday materials using an electronic balance
American Journal of Physics 85, 327 (2017); 10.1119/1.4975588

Klein Paradox for the Klein-Gordon Equation
American Journal of Physics 27, 355 (1959); 10.1119/1.1934851

Magnetic Susceptibility of Ferrous Ammonium Tutton Salt at Liquid Helium Temperatures
The Journal of Chemical Physics 29, 337 (1958); 10.1063/1.1744482
Equipment for the Determination of Magnetic Susceptibilities

J. A. McMillan
Instituto de Fisica de San Carlos de Bariloche, Bariloche, Rio Negro, Argentina
(Received October 20, 1958)

Equipment for the determination of magnetic susceptibilities by the Gouy method utilizing an electromagnet and a Westphal specific gravity balance is described. Using Mohr’s salt as a standard, the paramagnetism of salts like pentahydrated copper sulphate can be determined within 4%. The diamagnetism of some substances, such as water, can also be measured to one significant figure. Experimental results are given.

INTRODUCTION

Magnetic measurements are usually considered to be rather difficult, specially those dealing with para- and diamagnetism. Under such conditions, one is tempted to restrict the problem to the blackboard. Nevertheless, students can determine paramagnetic susceptibilities with the facilities available in a great many laboratories, e.g., a small electromagnet of about 7000 gauss and a Westphal balance. They can first calibrate the field by means of a standard paramagnetic, like Mohr’s salt, and then determine the susceptibilities of some compounds, such as cupric or manganous sulfates. They can even be allowed to observe the behavior of some diamagnetic substance, such as water, although they would not be able to determine more than one significant figure in the latter case. The method is fully described and discussed in the following text.

THE GOUY METHOD

The difference in magnetic potential energy per unit volume between a substance of permeability $\mu$ and the displaced medium—generally air—of permeability $\mu_0$ is

$$-(\mu-\mu_0/8\pi)H^2,$$

(1)

where $H$ is the intensity of the magnetic field. Taking into account that

$$\mu = 1 + 4\pi \chi; \quad \mu_0 = 1 + 4\pi \kappa_0,$$

(2)

where $\chi$ is the volume susceptibility, expression (1) transforms into

$$-\frac{1}{2}(\kappa-\kappa_0)H^2.$$

(3)

Now let us consider a cylindrical sample, with its axis normal to the field, of cross section $A$ and length $L$. The force acting on the volume element $A dz$ is

$$-A dz \frac{d}{dz} \left( \frac{\kappa-\kappa_0}{2} H^2 \right) = A (\kappa-\kappa_0) H dH,$$

(4)

whose integration from $z=0$ to $z=L$ leads to

$$f = (A/2)(\kappa-\kappa_0)(H^2-H_0^2),$$

(5)

where $H$ and $H_0$ are the values of the field intensity at $z=0$ and $z=L$, respectively. If the bottom end is placed in the maximum field at the middle of the pole pieces and $L$ is sufficiently long with respect to the diameter, the value of $H_0$ can be made small enough with respect to $H$ as to be neglected. Furthermore, the value of $\kappa_0$ is negligible when compared to $\kappa$ ($\kappa_0$ is approximately $0.03 \times 10^{-4}$ emu/cm³ at NTP in the case of air) at least within the scope of this paper, from which results

$$f = (A/2)\kappa H^2.$$

(6)

Although the volume susceptibility $\kappa$ has a very significant physical meaning, it is advantageous to introduce the specific susceptibility $\chi$ as

$$\kappa = \delta \chi,$$

(7)

where $\delta$ is the apparent density of the sample. Equation (6) then becomes

$$f = (MIH^2/2L)\chi,$$

(8)

where $M$ is the mass of the sample.

The foregoing derivations obviously imply the assumptions of a cross section sufficiently constant along the probe, a flat bottom end, and a homogeneous distribution of the powder in the cylinder.
The problem is to measure $f$, which is readily performed by weighing the sample with and without field. If $m$ is the measured difference, then

$$f = mg,$$

(9)

where $g$ is the acceleration of gravity, and the sign depends on the susceptibility, $m$ being positive for paramagnetic substances and negative for diamagnetic substances.

From Eqs. (8) and (9) one obtains

$$\chi = 2Lmg/MH^2.$$  

(10)

In order to determine $H$, it is desirable to use a known paramagnetic, easily purifiable and stable salt, such as FeSO$_4$·(NH$_4$)$_2$SO$_4$·6H$_2$O, also called Mohr's salt, which is commonly used as a standard, and whose specific susceptibility is $^{2}$

$$\chi_0 = 32.6 \times 10^{-8} \text{ emu/g at } 17^\circ\text{C}.$$  

From Eq. (10) the value of the field is

$$H = \left(\frac{2Lmg}{M\chi_0}\right)^{1/4}$$  

(11)

and eliminating $H$, $L$, and $g$ between Eqs. (10) and (11) gives

$$\chi = \frac{mM_0}{m_0M}$$  

(12)

where $\chi$ is the susceptibility of the unknown, its mass being $M$ and $m$ the observed difference.

Some care must be taken in controlling the temperature to within a few degrees, since the susceptibility of paramagnetic substances often depends strongly on it, following either one or another of the following laws:

$$\chi = (C/T) + \chi_0;$$  

(13)

$$\chi = (C/T - \theta) + \chi_0;$$  

(14)

$$\chi = (C/T + \Delta) + \chi_0;$$  

(15)

$C$, $\theta$, and $\Delta$ being the Curie, Weiss, and Néel positive constants, respectively.$^{3}$

Equation (13) holds for typical paramagnetism, Eq. (14) for low-temperature ferromagnetism, and Eq. (15) for antiferromagnetism, the latter two cases belonging to magnetically dense substances, the former to dilute ones.$^{2}$


$^{3}$ M. H. L. Pryce, Nuovo cimento 6, Suppl. 3, 817 (1957).

**FIG. 1. Apparatus for susceptibility measurements.**

**EXPERIMENTAL PROCEDURE**

The material needed for the experimental determinations is available in almost every laboratory—namely, (a) an electromagnet with flat pole pieces of about 30–35 mm diameter, separated about 10 mm, with which fields of not more than 7000 gauss are readily attained, and (b) a Westphal specific gravity balance, of the type of those used for rapid determinations of liquid densities up to 2 g/cm$^3$, that have a plummet of 5-g-water displacement and a sensitivity of about $10^{-4}$ g/cm$^3$.

Figure 1 is a photograph of the installation. The sample is suspended in the place of the plummet. It consists of a cylindrical Pyrex container, 10–12 cm long and 6-mm i.d., with a flat bottom end, wherein the powdered or the liquid sample is placed up to a level about twice the diameter of the pole pieces, to fulfill the requirement $H^2 \ll H^2$ assumed in Eq. (6). A perforated cork and an aluminum wire of 2-mm diameter for the suspension complete the device.

The bottom, flat end of the probe, must be placed near the middle of the pole pieces. Its position can be modified by means of the adjustable support of the balance, which can usually be clamped at any desired height between 20 and 30 cm.

It should be noted that if both the mass $M$ and the difference $m$ are measured in the conventional scale of the riders, as long as Eqs. (11) and (12) only include mass ratios, it is not necessary to know either the displacement of the plummet or the exact weight of the unit rider.

For purposes of accounting difference $m$, it is necessary to correct for the apparent loss in
weight due to the diamagnetism of the reservoir, which must then be previously determined, weighing the empty reservoir with and without field. This correction also includes any possible action of the outer magnetic field on the moving parts of the balance.

There follows a series of measurements made by us, where the masses are quoted in units of density.

**EXPERIMENTAL RESULTS**

**I. Calibration**

**Substance: powdered Mohr's salt**

\[
\begin{align*}
L &= 6.6 \pm 0.1 \text{ cm} \\
M_o &= 0.3780 \pm 0.0001 \\
n^* (\text{without diamagnetic correction}) &= 0.0390 \pm 0.0001 \\
\Delta n (\text{diamagnetic correction}) &= 0.0006 \pm 0.0001 \\
m (\text{corrected for diamagnetism}) &= 0.0396 \pm 0.0002.
\end{align*}
\]

From these results, one obtains, taking for \(x_0\) the value given before,

\[H = 6450 \pm 50 \text{ gauss.}\]

**II. Measurements**

**Substance: CuSO\(_4\):5H\(_2\)O, powdered,**

\[
\begin{align*}
L &= 6.6 \pm 0.1 \text{ cm} \\
M &= 0.4890 \pm 0.0001 \\
n^* &= 0.0089 \pm 0.0001 \\
\Delta n &= 0.0006 \pm 0.0001 \\
m &= 0.0935 \pm 0.0002 \\
H &= 6450 \pm 50 \text{ gauss.}
\end{align*}
\]

The observed value for \(\chi\) then results,

\[\chi_{\text{obs}} = (6.14 \pm 0.25) \times 10^{-6} \text{ emu/g},\]

while the value from tables is\(^3\)

\[\chi = 5.8 \times 10^{-6} \text{ emu/g.}\]

**Substance: liquid water,**

\[
\begin{align*}
L &= 6.6 \pm 0.1 \text{ cm} \\
M &= 0.3953 \pm 0.0001 \\
n^* &= -0.0016 \pm 0.0001 \\
\Delta n &= 0.0006 \pm 0.0001 \\
m &= -0.0010 \pm 0.0002 \\
H &= 6450 \pm 50 \text{ gauss.}
\end{align*}
\]

The observed value for \(\chi\) then results,

\[\chi_{\text{obs}} = -(0.80 \pm 0.16) \times 10^{-4} \text{ emu/g,}\]

while the value from tables is\(^4\)

\[\chi = -0.72 \times 10^{-4} \text{ emu/g.}\]

From these results, it is evident that a student can obtain training in better and more accurate techniques, and rather good results, even if not yet sufficiently skilled to use delicate and expensive equipment.


---

**Plasma Dynamics**

The Massachusetts Institute of Technology, through its Special Summer Program, will present a unified treatment of the fundamentals of plasma physics during the period August 17 through September 4, 1989. In addition, the Program will survey the present status of a large fraction of the unclassified research efforts in plasma dynamics. The fundamentals of the following subjects will be discussed: collision theory, Boltzmann transport theory, charged particle interactions with electric and magnetic fields, magnetohydrodynamics, thermonuclear machines, ionic propulsion, shock tubes, electromagnetic propagation, and plasma diagnostics. The general lectures will be supplemented with demonstrations and group discussions.

The Program will be directed by Professor William P. Allis of the Department of Physics. Professors Sanborn C. Brown of the Department of Physics, Osman K. Mawardi of the Department of Mechanical Engineering, David J. Rose of the Department of Nuclear Engineering, David R. Whitehouse of the Department of Electrical Engineering, and other members of the MIT faculty will participate. Another Program, "Organization for Research and Development," is intended for those responsible for directing research programs and supervising research engineers and scientists in industrial laboratories. For information on either Program, write to Professor James M. Austin, Director of the Summer Session, Massachusetts Institute of Technology, Cambridge 39, Massachusetts.
## Units for Magnetic Properties

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Gaussian &amp; cgs emu</th>
<th>Conversion factor, C</th>
<th>SI &amp; rationalized mks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic flux density, magnetic induction</td>
<td>B</td>
<td>gauss (G)</td>
<td>$10^{-4}$</td>
<td>tesla (T), Wb/m²</td>
</tr>
<tr>
<td>Magnetic flux</td>
<td>Φ</td>
<td>maxwell (Mx), G-cm²</td>
<td>$10^{-8}$</td>
<td>weber (Wb), volt second (V-s)</td>
</tr>
<tr>
<td>Magnetic potential difference, magnetomotive force</td>
<td>U, F</td>
<td>gilbert (Gb)</td>
<td>$10/4π$</td>
<td>ampere (A)</td>
</tr>
<tr>
<td>Magnetic field strength, magnetizing force</td>
<td>H</td>
<td>oersted (Oe), Gb/cm</td>
<td>$10^{3}/4π$</td>
<td>A/m °</td>
</tr>
<tr>
<td>(Volume) magnetization</td>
<td>M</td>
<td>emu/cm³</td>
<td>$10^3$</td>
<td>A/m °</td>
</tr>
<tr>
<td>(Volume) magnetization</td>
<td>4πM</td>
<td>G</td>
<td>$10^{3}/4π$</td>
<td>A/m °</td>
</tr>
<tr>
<td>Magnetic polarization, intensity of magnetization</td>
<td>J, I</td>
<td>emu/cm³</td>
<td>$4π×10^{-4}$</td>
<td>T, Wb/m² i</td>
</tr>
<tr>
<td>(Mass) magnetization</td>
<td>σ, M</td>
<td>emu/g</td>
<td>1</td>
<td>A-m²/kg</td>
</tr>
<tr>
<td>Magnetic moment</td>
<td>m</td>
<td>emu, erg/G</td>
<td>$10^{-3}$</td>
<td>A-m², joule per tesla (J/T)</td>
</tr>
<tr>
<td>Magnetic dipole moment</td>
<td>j</td>
<td>emu, erg/G</td>
<td>$4π×10^{-10}$</td>
<td>Wb-m i</td>
</tr>
<tr>
<td>(Volume) susceptibility</td>
<td>χ, κ</td>
<td>dimensionless, emu/cm³</td>
<td>$4π$ $(4π^2×10^{-7}$</td>
<td>dimensionless henry per meter (H/m), Wb/(A-m)</td>
</tr>
<tr>
<td>(Mass) susceptibility</td>
<td>χₚ, ρ</td>
<td>cm³/g, emu/g</td>
<td>$4π×10^{-3}$</td>
<td>m³/kg</td>
</tr>
<tr>
<td>(Molar) susceptibility</td>
<td>χₚₒ, ρₒ</td>
<td>cm³/mol, emu/mol</td>
<td>$4π×10^{-6}$</td>
<td>m³/mol</td>
</tr>
<tr>
<td>Permeability</td>
<td>μ</td>
<td>dimensionless</td>
<td>$4π×10^{-7}$</td>
<td>H/m, Wb/(A-m)</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>μᵣ</td>
<td>not defined</td>
<td>1</td>
<td>dimensionless</td>
</tr>
<tr>
<td>(Volume) energy density, energy product</td>
<td>W</td>
<td>erg/cm³</td>
<td>$10^{-1}$</td>
<td>J/m³</td>
</tr>
<tr>
<td>Demagnetization factor</td>
<td>D, N</td>
<td>dimensionless</td>
<td>$1/4π$</td>
<td>dimensionless</td>
</tr>
</tbody>
</table>

*a*. Gaussian units and cgs emu are the same for magnetic properties. The defining relation is $B = H + 4πM$.

*b*. Multiply a number in Gaussian units by $C$ to convert it to SI (e.g., $1$ G $× 10^{-4}$ T/G $= 10^{-4}$ T).

*c*. SI (Système International d'Unités) has been adopted by the National Bureau of Standards. Where two conversion factors are given, the upper one is recognized under, or consistent with, SI and is based on the definition $B = μ₀(H + M)$, where $μ₀ = 4π × 10^{-7}$ H/m. The lower one is not recognized under SI and is based on the definition $B = μH + J$, where the symbol $J$ is often used in place of $J$.

*d*. 1 gauss $= 10^5$ gamma ($γ$).

*e*. Both oersted and gauss are expressed as cm$^{-1/2}$-g$^{1/2}$-s$^{-1}$ in terms of base units.

*f*. A/m was often expressed as “ampere-turn per meter” when used for magnetic field strength.

*g*. Magnetic moment per unit volume.

*h*. The designation “emu” is not a unit.

*i*. Recognized under SI, even though based on the definition $B = μH + J$. See footnote c.

*j*. $μᵣ = μ/μ₀ = 1 + χ$, all in SI. $μᵣ$ is equal to Gaussian $μ$.

*k*. $B·H$ and $μ₀M·H$ have SI units J/m³; $M·H$ and $B·H/4π$ have Gaussian units erg/cm³.

---

Appendix 5

CHARLES KITTEL

Introduction

to

Solid State

Physics

THIRD EDITION

14

Diamagnetism
and
Paramagnetism

Measurement of susceptibility 429

LANGEVIN DIAMAGNETISM EQUATION 430
Diamagnetism of molecules 431

PARAMAGNETISM 432

LANGEVIN PARAMAGNETISM EQUATION AND THE CURIE LAW 432

QUANTUM THEORY OF PARAMAGNETISM 434
Rare earth ions 437
Iron group ions 438
Crystal field splitting 438
Quenching of the orbital angular momentum 439
Nuclear paramagnetism 440

COOLING BY ADIABATIC DEMAGNETIZATION OF A PARAMAGNETIC SALt 440
Example 443
Nuclear demagnetization 444

PARAMAGNETIC SUSCEPTIBILITY OF CONDUCTION ELECTRONS 446

PROBLEMS 449

REFERENCES 451
This is the ferromagnetism resonance.

The magnetic moment. The magnetic moment $m$ here is the magnitude of the magnetic moment, which may be ferromagnetic for certain substances. Substances with ordered arrays of magnetic moments may be ferromagnetic.

Measurement of Magnetic Moment

The magnetic moment of a specimen is given by $m = \frac{1}{2} \chi H^2$, where $\chi$ is the susceptibility of the material and $H$ is the applied field.

In the Gouy method, the specimen is placed halfway into a static magnetic field, and the gradient of the field is provided.

---

Figure 1: Characteristic magnetic susceptibilities of diamagnetic and paramagnetic substances.

Figure 2: (a) Faraday and (b) Gouy methods for measuring magnetic susceptibility. The magnet pole pieces in the Faraday method are designed to produce high values of $H$ at the specimen. Values of $10^{-6}$ T cm are found in good commercial equipment, so that the force on $1 \text{ cm}^3$ of a specimen having $\chi = 10^{-6}$ is 10 dynes. In the Gouy method a high value of $H$ is desired.
This is the first of three chapters on magnetism. Chapter 15 treats ferromagnetism and antiferromagnetism, and Chapter 16 treats magnetic resonance.

The magnetization $M$ is defined as the magnetic moment per unit volume. The magnetic susceptibility per unit volume is defined as

$$\chi = \frac{M}{H};$$

(1)

here $H$ is the macroscopic magnetic field intensity. In Gaussian units $\chi$ is dimensionless. Quite frequently a susceptibility is defined referred to unit mass or to a mole of the substance. The molar susceptibility is written as $\chi_M$; the magnetic moment per gram is sometimes written as $\sigma$.

Substances with a negative magnetic susceptibility are called diamagnetic. Substances with a positive susceptibility are called paramagnetic, as in Fig. 1. Ordered arrays of magnetic moments are discussed in Chapter 15; the arrays may be ferromagnetic, ferrimagnetic, antiferromagnetic, helical, or more complex in form. Nuclear magnetic moments give rise to nuclear paramagnetism. Magnetic moments of nuclei are of the order of $10^{-3}$ times smaller than the magnetic moment of the electron.

Measurement of Susceptibility

The magnetic contribution to the energy density of a magnetized specimen is $\frac{1}{2} \chi H^2$, provided $\chi$ is independent of $H$. The force on a unit volume is the gradient of the energy density. The $x$ component of the force on a small specimen of volume $V$ is

$$F_x = \frac{1}{2} \chi V \frac{d}{dx} H^2 = \chi VH \cdot (dH/dx),$$

(2)

provided the field $H$ and the derivative $dH/dx$ do not vary appreciably over the volume (this is why the specimen must be small). Equation (2) is the basis of the Faraday method (Fig. 2a) which is used with small samples and a sensitive microbalance. The sensitivity of the Faraday method is high: a susceptibility change of $10^{-10}$ may be measured.

In the Gouy method (Fig. 2b) a long cylindrical sample is suspended halfway into a strong field $H$: one end of the sample is in the maximum field and the other end is in a region where the field is negligible. The total force

\[1\text{ We could well write } B \text{ in place of } H \text{ for the magnetic field intensity in the absence of the specimen, but our usage of } H \text{ is established in the literature of magnetism. We shall express both } B \text{ and } H \text{ in gauss (G) or kilogauss (kG). The matter of names and symbols in magnetism is treated definitively in Chap. 10 of E. M. Purcell, Electricity and magnetism, Berkeley physics course, Vol. 2, McGraw-Hill, 1965.}\]
under these conditions on a specimen of cross-sectional area $A$ is

$$F_z = \frac{1}{2} x A \int dx \frac{d}{dx} H^2 = \frac{1}{2} x A H^2.$$  

(3)

**LANGEVIN DIAMAGNETISM EQUATION**

Diamagnetism is associated with the tendency of electrical charges partially to shield the interior of a body from an applied magnetic field. In electromagnetism we are familiar with Lenz’s law, which states that when the flux through an electrical circuit is changed, an induced current is set up in such a direction as to oppose the flux change. In a resistanceless circuit, in a superconductor, or in an electron orbit within an atom, the induced current persists as long as the field is present. The magnetic field produced by the induced current is opposite to the applied field, and the magnetic moment associated with the current is a diamagnetic moment.

The usual treatment of the diamagnetism of atoms and ions employs the Larmor theorem, which states that in a magnetic field the motion of the electrons around a central nucleus is, to the first order in $H$, the same as a possible motion in the absence of $H$ except for the superposition of a precession of angular frequency

$$\omega = eH/2mc.$$  

(4)

If the field is applied slowly, the motion in the rotating reference system will be the same as the original motion in the rest system before the application of the field. If the average electron current around the nucleus was zero initially, then the application of the magnetic field will cause a finite average current around the nucleus. The current thus established is equivalent to a magnetic moment. The direction of the moment is opposite to the direction of the applied field.

The Larmor precession of the electron distribution is equivalent to an electric current

$$I = -(Ze)(eH/2mc)/2mc.$$  

(5)

in electromagnetic units. The magnetic moment $\mu$ of a current loop is given by the product of the current (in emu) times the area of the loop. The area of a loop of radius $\rho$ is $\pi \rho^2$. We have

$$\mu = -(Ze^2H/4mc^2)\langle \rho^2 \rangle,$$  

(6)

for $Z$ electrons; here $\langle \rho^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle$ is the average of the square of the perpendicular distance of the electron from the field axis.

The mean square distance of the electrons from the nucleus is $\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$. For a distribution of charge which is spherically sym-

metrical $\langle x^2 \rangle = \chi_m$ in $10^{-6}$

This is the class in Appendix F.

The problem atom is reduced the atom. The mental values |

Typical experin

Diamagnetism

The deriv direction is an condition is no must be appl we have, acco

where $N_0$ is the $z$ component $\epsilon$ with the excit states. The ma first or second paramagnetism

For the n

$Xn$

cm$^3$/mole. Th


$^3$ J. H. Van V
PARAMAGNETISM

Electronic paramagnetism (positive contribution to $\chi$) is found in:

a. Atoms, molecules, and lattice defects possessing an odd number of electrons, as here the total spin of the system cannot be zero. Examples: free sodium atoms; gaseous nitric oxide (NO); organic free radicals such as triphenylmethyl, C(C$_6$H$_5$)$_3$; $F$ centers in alkali halides.

b. Free atoms and ions with a partly filled inner shell; transition elements; ions isoelectronic with transition elements; rare earth and actinide elements. Examples: Mn$^{2+}$, Gd$^{3+}$, U$^{4+}$. Paramagnetism is exhibited by many of these ions when incorporated into solids, but not invariably.

c. A few compounds with an even number of electrons, including molecular oxygen and organic biradicals.

d. Metals.

We shall consider classes (b) and (d).

LANGEVIN PARAMAGNETISM EQUATION AND THE CURIE LAW

We treat a medium containing $N$ atoms per unit volume, each bearing a magnetic moment $\mu$. Magnetization results from the orientation of the magnetic moments in an applied magnetic field; thermal disorder resists the tendency of the field to orient the moments. The energy of interaction with an applied magnetic field $H$ is

$$U = -\mu \cdot H.$$  \hfill (10)

The magnetization in thermal equilibrium is calculated by following exactly the steps (12.25) to (12.28) in the derivation of the Debye orientational polarizability, with $\mu$ written for $p$ and $H$ for $E$. The magnetization is then given by the Langevin equation

$$M = N\mu L(x),$$  \hfill (11)

where $x \equiv \mu H/k_B T$, and the Langevin function $L(x)$ is

$$L(x) \equiv \text{ctnh} \ x - \frac{1}{x}.$$  \hfill (12)

For $x \ll 1$, we have $L(x) \approx x/3$ from (12.29), so that the magnetization is

$$M \approx N\mu^3 H/3k_B T.$$  \hfill (13)

For an electron in a spherical sample of gadolinium sulfate, the magnetization is 50,000 gauss. [Afu 88,559 (1952).]
number of elements; including trivalent elements; such as trivalent elements; one of these being exactly orientational ion is then

\begin{align*}
10) \\
(11) \\
(12) \\
(13)
\end{align*}

Figure 3  Plot of magnetic moment versus $H/T$ for spherical samples of (I) potassium chromium alum, (II) ferric ammonium alum, and (III) gadolinium sulfate octahydrate. Over 90.5% magnetic saturation is achieved at 1.3°K and about 50,000 gauss. [After W. E. Henry, Phys. Rev. 88, 559 (1952).]

For an electron $\mu \approx 10^{-20}$ erg/gauss. At room temperature in a field of 10 gauss we have $\mu H/k_B T \approx 2 \times 10^{-3}$, so that under these conditions we may safely approximate the Langevin function by $\mu H/3k_B T$. At low temperatures saturation effects have been observed, as shown in Fig. 3.

The magnetic susceptibility in the limit $\mu H/k_B T \ll 1$ is

\begin{equation}
\chi = \frac{M}{H} = \frac{N\mu^2}{3k_B T} = \frac{C}{T},
\end{equation}

where the Curie constant $C = N\mu^2/3k_B$. The result (14) is known as the Curie law.
## 15

**Ferromagnetism**

*and*

**Antiferromagnetism**

<table>
<thead>
<tr>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ferromagnetic Order</td>
<td>455</td>
</tr>
<tr>
<td>Curie point and the exchange integral</td>
<td>455</td>
</tr>
<tr>
<td>Temperature dependence of the saturation magnetization</td>
<td>458</td>
</tr>
<tr>
<td>Saturation magnetization at absolute zero</td>
<td>461</td>
</tr>
<tr>
<td>Spin waves</td>
<td>464</td>
</tr>
<tr>
<td>Quantization of spin waves</td>
<td>467</td>
</tr>
<tr>
<td>Thermal excitation of magnons</td>
<td>468</td>
</tr>
<tr>
<td>Neutron scattering: elastic and inelastic</td>
<td>469</td>
</tr>
<tr>
<td>Ferrimagnetic Order</td>
<td>472</td>
</tr>
<tr>
<td>Curie temperature and susceptibility of ferrimagnets</td>
<td>477</td>
</tr>
<tr>
<td>Iron garnets</td>
<td>478</td>
</tr>
<tr>
<td>Antiferromagnetic Order</td>
<td>480</td>
</tr>
<tr>
<td>Susceptibility below the Néel temperature</td>
<td>484</td>
</tr>
<tr>
<td>Antiferromagnetic magnons</td>
<td>485</td>
</tr>
<tr>
<td>Helical order</td>
<td>486</td>
</tr>
<tr>
<td>Ferromagnetic Domains</td>
<td>488</td>
</tr>
<tr>
<td>Anisotropy energy</td>
<td>490</td>
</tr>
<tr>
<td>Transition region between domains</td>
<td>493</td>
</tr>
<tr>
<td>Origin of domains</td>
<td>494</td>
</tr>
<tr>
<td>Coercive force and hysteresis</td>
<td>495</td>
</tr>
<tr>
<td>Problems</td>
<td>497</td>
</tr>
<tr>
<td>References</td>
<td>498</td>
</tr>
</tbody>
</table>
FERROMAGNETIC ORDER

A ferromagnet has a spontaneous magnetic moment—a magnetic moment even in zero applied magnetic field. A spontaneous moment suggests that electron spins and magnetic moments are arranged in a regular manner. The order need not be simple; all of the spin arrangements sketched in Fig. 1 except the simple antiferromagnet (and except the helix if the spin directions lie in a plane) have a spontaneous magnetic moment, usually called the saturation moment.

Curie Point and the Exchange Integral

Consider a paramagnet with a concentration of \( N \) ions of spin \( S \). Given an internal interaction tending to line up the magnetic moments parallel to each other, we shall have a ferromagnet. Let us postulate such an interaction and call it the exchange field or molecular field.\(^1\) The orienting effect of the exchange field is opposed by thermal agitation, and at elevated temperatures the spin order is destroyed.

We treat the exchange field as equivalent to a magnetic field \( H_E \). The magnitude of the exchange field may be as high as \( 10^7 \) gauss. We assume further that \( H_E \) is proportional to the magnetization \( M \). The magnetization \( M(H, T) \) is defined as the magnetic moment per unit volume; unless otherwise specified it is understood to be the value in thermal equilibrium in the field \( H \) at the temperature \( T \). If domains (regions magnetized in different directions) are present, the magnetization refers to the value within a domain. Thus we assume

\[
H_E = \lambda M, \tag{1}
\]

where \( \lambda \) is a constant, independent of temperature. According to (1) each spin sees the average magnetization of all the other spins. Actually, it may see only near neighbors, but our oversimplification of the situation is good for a first look at the problem.

The Curie temperature \( T_c \) is the temperature above which the spontaneous magnetization vanishes; it separates the disordered paramagnetic phase at \( T > T_c \) from the ordered ferromagnetic phase at \( T < T_c \). We can find \( T_c \) in terms of \( \lambda \). Consider the paramagnetic phase; an applied field \( H \) will cause a finite magnetization and this in turn will cause a finite exchange field \( H_E \). If \( \chi \) is the susceptibility,

\[
M = \chi(H + H_E). \tag{2}
\]

\(^1\) Also called the Weiss field, after Pierre Weiss who was the first to imagine such a field.
We are now in the position to understand why the Circle is the least expensive of the three models (5) and (6).

\[ S = \frac{1}{2} J \]

The Circle is the least expensive of the three because it has the least variance in the cost of the exchange rate.

\[ \text{Var}(\text{Exchange Rate}) = \text{Var}(\text{Circle}) < \text{Var}(\text{Square}) < \text{Var}(\text{Triangle}) \]

The Circle is the least expensive because it has the least variance in the cost of the exchange rate. This is due to the Circle's smooth and constant nature, which results in a more stable and predictable exchange rate.

We see in Figure 14 that the Circle reduces the variance in the cost of the exchange rate compared to the Square and Triangle models.

\[ \text{Var}(\text{Circle}) < \text{Var}(\text{Square}) < \text{Var}(\text{Triangle}) \]

The Circle is the most stable and predictable of the three models. The Square and Triangle models have higher variance in the cost of the exchange rate, which results in more fluctuations and unpredictability.

\[ \text{Var}(\text{Exchange Rate}) = \text{Var}(\text{Square}) > \text{Var}(\text{Triangle}) \]

We see in Figure 15 that the Circle is the most stable and predictable model, while the Square and Triangle models have higher variance in the cost of the exchange rate.

\[ \text{Var}(\text{Circle}) < \text{Var}(\text{Square}) < \text{Var}(\text{Triangle}) \]

The Circle is the best model for predicting the cost of the exchange rate. The Square and Triangle models have higher variance in the cost of the exchange rate, which results in more fluctuations and unpredictability.

\[ \text{Var}(\text{Exchange Rate}) = \text{Var}(\text{Square}) > \text{Var}(\text{Triangle}) \]

We see in Figure 16 that the Circle is the most stable and predictable model, while the Square and Triangle models have higher variance in the cost of the exchange rate.

\[ \text{Var}(\text{Circle}) < \text{Var}(\text{Square}) < \text{Var}(\text{Triangle}) \]

The Circle is the best model for predicting the cost of the exchange rate. The Square and Triangle models have higher variance in the cost of the exchange rate, which results in more fluctuations and unpredictability.

\[ \text{Var}(\text{Exchange Rate}) = \text{Var}(\text{Square}) > \text{Var}(\text{Triangle}) \]
ion as a constant $\chi$

the assumption of a susceptibility tensor constant. Using

\begin{equation}
(3)
\end{equation}

temperature (and $T$ is infinite) we can use Veiss law

\begin{equation}
(4)
\end{equation}

eptibility variation calculations\(^2\) predict also favor the nickel are shown in constant $C$ we may

\begin{equation}
(5)
\end{equation}

we have $\lambda \approx 5000$, $\approx 10^7$ G. The exchange field depends on the quantum; it can be shown\(^4\) $S_z$ contains a term

\begin{equation}
(6)
\end{equation}

clap of the charge Heisenberg model.

by $G(T - \theta)$, where $\theta$ is a review by G. Domb. For experimental ref.\(^5\), in the metal is close to $1$ in Phys. 17, 27 (1945).

\begin{itemize}
  \item crystals. The origin of $\lambda$, Vol. 1, 23 (1963); \textit{Ferromagnetism and Antiferromagnetism} 457
\end{itemize}

The exchange energy has no classical analog, but the energy is of electrostatic origin. It expresses the difference in Coulomb interaction energy of the systems when the electron spins are parallel or antiparallel, as in Fig. 3.13. Because of the Pauli exclusion principle we cannot change the relative direction of two spins without changing the spatial distribution of charge. The resulting changes in the Coulomb energy of the system may be written in the form\(^6\) (6), as if there were a direct coupling between the directions of the spins $S_i, S_j$.

We establish an approximate connection between the exchange integral $J$ in (6) and the exchange constant $\lambda$ in (1). We suppose that the atom under consideration has $z$ nearest neighbors, each connected with the central atom by the interaction $J$. For more distant neighbors we take $J$ as zero. The energy required to reverse the spin under consideration in the presence of all other spins may be written, neglecting components of $S$ perpendicular to the average magnetization, as

\begin{itemize}
  \item If two spins are parallel, the spatial part of the wavefunction must be antisymmetric under the exchange of the two electrons. If the spins are antiparallel, the spatial part of the wavefunction is symmetric.
  \item Equation (6) is an equation in the spin operators $S_i, S_j$ for many purposes in ferromagnetism it is a good approximation to treat the spins as classical angular momentum vectors.
\end{itemize}
\[ U = 4JzS^z = 2\mu H_E = 2\mu(\lambda M_s) = 2\mu(\lambda\mu/\Omega) \]  

(7)

where \( \bar{S} \) is the average value of \( S \) in the direction of the magnetization and \( \Omega \) is the volume per atom.

The average magnetic moment of a spin is \( \mu = g\bar{S}\mu_B \), and the saturation magnetization is \( M_s = \mu/\Omega \), Thus from (7)

\[ \lambda = 2Jz\Omega/g^2\mu_B^2, \]  

(8)

where \( z \) is the number of nearest neighbors.

Using (5) and \( \Omega = 1/N \), we have the molecular field theory result:

\[ J = \frac{3k_B T_c}{2zS(S + 1)}. \]  

(9)

Better approximations to the quantum-statistical problem give somewhat different results for \( 3J/k_B T_c \). For the \( sc \), \( bcc \), and \( fcc \) structures with \( S = \frac{1}{2} \), Rushbrooke and Wood\(^1\) give \( k_B T_c/3J = 0.28; 0.325; \) and 0.346, respectively, as compared with 0.500 from (9) for all three structures. If iron is represented by the Heisenberg model (6) with \( S = 1 \), then the observed Curie temperature corresponds to \( J = 1.19 \times 10^{-2} \) ev.

**Temperature Dependence of the Saturation Magnetization**

We can also use the molecular field approximation below the Curie temperature to find the magnetization as a function of temperature. We proceed as before, but instead of the Curie law we use the complete Brillouin expression (14.23) for the magnetization. For spin \( \frac{1}{2} \) this is\(^8\) \( M = N\mu \tanh(\mu H/k_BT) \), according to (14.20). If we omit the applied magnetic field and replace \( H \) by the molecular field \( H_E = \lambda M \), then

\[ M = N\mu \tanh(\mu\lambda M/k_BT). \]  

(10)

We shall see that solutions of this equation with nonzero \( M \) exist in the temperature range between 0 and \( T_c \).

To solve (10) we write it in parametric form as

\[ M = N\mu \tanh(\xi); \quad \mu\lambda M/k_BT = \xi, \]  

(11)

and plot \( M \) versus \( \xi \) for both equations. The intercept of the two curves gives the value of \( M \) at the temperature of interest, as in Fig. 3. As \( M \to 0 \), we have \( M \to N\mu(\mu\lambda M/k_BT_c) \), or \( T_c = \lambda(N\mu^2/k_B) \), in agreement with (5) for \( S = \frac{1}{2} \).

The curves of \( M \) versus \( T \) obtained in this way reproduce roughly the features of the experimental results, as shown in Fig. 4 for nickel. As \( T \)

\(^1\) G. S. Rushbrooke and P. J. Wood, Molecular Physics 1, 957 (1958).

\(^8\) We often write \( M_s \) for the spontaneous or saturation magnetization, but where no ambiguity is possible we shall use \( M \). Compare Fig. 37.
ANTIFERROMAGNETIC ORDER\textsuperscript{27}

A classical example of magnetic structure determination by neutrons is shown in Fig. 26 for MnO, which has the NaCl structure. At 80°K there are extra neutron reflections not present at 293°K. The reflections at 80°K may be classified in terms of a cubic unit cell of lattice constant 8.85 Å. At 293°K the reflections correspond to an fcc unit cell of lattice constant 4.43 Å. But the lattice constant determined by x-ray reflection is 4.43 Å at both temperatures, 80°K and 293°K. We conclude that the chemical unit cell has the 4.43 Å lattice parameter, but that at 80°K the electronic magnetic moments of the Mn\textsuperscript{4+} ordering the same as with the net spins in a six parallel. Thus

\textsuperscript{27} Data on antiferromagnetic structures studied by neutron diffraction are given in Table 5g-22 of the A.I.P. Handbook, McGraw-Hill, 1963, 3rd ed.

\textsuperscript{28} Mn\textsuperscript{2+} l.
Ordered arrangement of spins of the Mn$^{++}$ ions in manganese oxide, MnO, as determined by neutron diffraction. The O$^{--}$ ions are not shown.

Spin ordering in ferromagnets ($J > 0$) and antiferromagnets ($J < 0$).

... ordering... Phys. Rev. and on a netic, but... by neutrons is... at 80°K there are... ns at 80°K may... 85 Å. At 293°K... taut 4.43 Å. But at both tempe... unit cell has the... magnetic moments... are given in Table... of the Mn$^{++}$ ions$^{28}$ are ordered in some nonferromagnetic arrangement. If the ordering were ferromagnetic, the chemical and magnetic cells would give the same reflections. The spin arrangement shown in Fig. 27 is consistent with the neutron diffraction results and with magnetic measurements. The spins in a single [111] plane are parallel, but adjacent [111] planes are antiparallel. Thus MnO is an antiferromagnet, as in Fig. 28.

In an antiferromagnet the spins are ordered in an antiparallel arrangement with zero net moment at temperatures below the ordering or Néel...

$^{28}$ Mn$^{++}$ has a magnetic moment of 5μB.
temperature. The susceptibility of an antiferromagnet is not infinite at $T = T_N$, but has a weak cusp, as in Figs. 29 and 30.

An antiferromagnet is a special case of a ferrimagnet for which both sublattices $A$ and $B$ have equal saturation magnetizations. Thus $C_A = C_B$ in (47), and the Néel temperature is given by

$$T_N = \mu C,$$

where $C$ refers to a single sublattice. The susceptibility in the paramagnetic region $T > T_N$ is obtained from (49):

$$X = \frac{2C}{T^2 - (\mu C)^2} = \frac{2C}{T + \mu C} = \frac{2C}{T + T_N}.$$  

The experimental results at $T > T_N$ are of the form

$$X = \frac{2C}{T + \theta}.$$  

Experimental values of $\theta/T_N$ listed in Table 2 often differ substantially from the value unity expected from (51). Values of $\theta/T_N$ of the observed magnitude may be obtained when next-nearest-neighbor interactions are provided for, and when general sublattice arrangements are considered. It is shown in Problem 3 that if a molecular field constant $-\epsilon$ is introduced to describe interactions within a sublattice, then $\theta/T_N = (\mu + \epsilon)/(\mu - \epsilon)$.


is not infinite at

or which both sub-

$C_d = C_B$ in (47),

(50)

the paramagnetic

(51)

substantially from

observed magni-

ons\(^{29}\) are provided. It is shown

duced to describe

- $\epsilon$.

s. Rev. 81, 1015 (1951).

Table 2  Summary of antiferromagnetic data

The Néel temperatures $T_N$ often vary considerably between samples, and in some cases there is large thermal hysteresis. For a bibliography relating to experimental data on antiferromagnetic substances, see T. Nagamiya, K. Yosida, and R. Kubo, Advances in

Physics 4, 1-112 (1955); and the A.I.P. Handbook. The value of $\theta$ is obtained by fitting an expression of the form $\chi = C/(T + \theta)$ to the susceptibility above the actual transition temperature $T_N$.

<table>
<thead>
<tr>
<th>Substance</th>
<th>Paramagnetic lattice</th>
<th>$T_N$ in °K</th>
<th>Curie-Weiss $\theta$ in °K</th>
<th>$\theta / T_N$</th>
<th>$\chi(0)$</th>
<th>$\chi(T_N)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MnO</td>
<td>fcc</td>
<td>116</td>
<td>610</td>
<td>5.3</td>
<td>$\frac{2}{3}$</td>
<td></td>
</tr>
<tr>
<td>MnS</td>
<td>fcc</td>
<td>160</td>
<td>523</td>
<td>3.3</td>
<td>0.82</td>
<td></td>
</tr>
<tr>
<td>MnTe</td>
<td>hex. layer</td>
<td>307</td>
<td>690</td>
<td>2.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MnF$_2$</td>
<td>bc tetr</td>
<td>67</td>
<td>82</td>
<td>1.24</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td>FeF$_2$</td>
<td>bc tetr</td>
<td>79</td>
<td>117</td>
<td>1.48</td>
<td>0.72</td>
<td></td>
</tr>
<tr>
<td>FeCl$_2$</td>
<td>hex. layer</td>
<td>24</td>
<td>48</td>
<td>2.0</td>
<td>$&lt;0.2$</td>
<td></td>
</tr>
<tr>
<td>FeO</td>
<td>fcc</td>
<td>198</td>
<td>570</td>
<td>2.9</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>CoCl$_2$</td>
<td>hex. layer</td>
<td>25</td>
<td>38.1</td>
<td>1.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CoO</td>
<td>fcc</td>
<td>291</td>
<td>330</td>
<td>1.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NiCl$_2$</td>
<td>hex. layer</td>
<td>50</td>
<td>68.2</td>
<td>1.37</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NiO</td>
<td>fcc</td>
<td>525</td>
<td>$-2000$</td>
<td>$-4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$-Mn</td>
<td>complex</td>
<td>$\sim 100$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cr</td>
<td>bcc</td>
<td>308</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CrSb</td>
<td>hex. layer</td>
<td>723</td>
<td>550</td>
<td>0.76</td>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>Cr$_2$O$_3$</td>
<td>complex</td>
<td>307</td>
<td>485</td>
<td>1.58</td>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
<tr>
<td>FeCo$_2$</td>
<td>complex</td>
<td>35</td>
<td>14</td>
<td>0.4</td>
<td>$\frac{1}{2}$</td>
<td></td>
</tr>
</tbody>
</table>