COSMIC RAY MUONS

v2.6
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I. INTRODUCTION

The goal of this experiment is to study two important properties of cosmic ray muons. The first concern is the study of the radioactive decay and the measurement of the mean life of this fundamental subatomic particle with high precision (< 1%). The second concern is the measurement of the flux of these particles at sea level.

The apparatus and techniques used in the experiment are common to many experiments in high energy and nuclear physics, so a subsidiary goal is to become familiar with these techniques. In addition, the data analysis involves the extensive use of the least-squares fitting technique to extract from the data the muon mean life and sea level flux.

II. COSMIC RAY MUONS

"Primary" cosmic rays are mainly protons and other nuclides which are generated in other parts of our galaxy. When they reach the earth's outer atmosphere, at an altitude of roughly 15 km, they interact with nuclei in the air and initiate a shower of nuclear and electromagnetic interactions. Many types of particles are produced: pions (π^+, π^-, π°), electrons, positrons, photons, kaons, protons, antiprotons, and others. The charged pions will interact by means of the strong force, but some of them decay by the weak force into muons and neutrinos. Muons do not interact via the strong nuclear force, but they do lose energy due to Coulomb scattering, and eventually they decay via the weak force into electrons and neutrinos. At sea level the "secondary" cosmic rays we can detect are mostly muons and electrons, as well as neutrons. High altitude observations and detailed theoretical analysis shows that the mean altitude for the formation of the muons we can detect at sea level is 15 km. If they were traveling at the speed of light, the trip would take 50 μsec. Given that their mean lifetime is about 2 μsec when they decay at rest, one has evidence from a comparison of high and low altitude flux measurements that the high-speed muons experience massive time dilation, as given by Special Relativity.

The muon flux which reaches sea level has a spectrum extending out to many GeV of kinetic energy. Most muons that enter the detector used in this experiment will pass through it, but those with kinetic energy less than 150 MeV lose enough energy by ionization to *stop* in the detector and subsequently decay at rest. It is mainly those that stop and decay which we study in this experiment.

The graph in Fig. 1 shows the distribution of sea level cosmic ray muons and other particle types as a function of their kinetic energy. We see that the most probable energy of the muons at sea level is about 500 MeV; the mean energy is about ~4000 MeV or ~4 GeV. For kinetic energies on the order of 100 MeV, the positive and negative muon fluxes are approximately equal. At higher energies, there is a positive excess. The positive and negative components of cosmic ray muons are discussed in Sec. 7.6 of Weissenberg. The highest flux is from straight up, and the intensity falls off approximately as $\cos^2\theta$, where θ is the angle of the incoming muon with respect to the vertical.

Exercise 1: *Look up the expected total flux of cosmic rays at sea level. The instructor can suggest where to look.*

The muon is one of the three known types of charged *leptons*: electrons, muons, and taus. It is unstable and decays as follows:

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu \quad \text{and} \quad \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \qquad (1)$$

The decay electron energy spectrum is given in Fig. 2. The energy of the electrons comes from the mass-energy of the decaying muons, but since there are three particles in the final state the electrons share the total available energy with the neutrinos.

The muon rest mass is 105.658369 ± 0.000009 MeV/c^2 and its mean life is currently known to be 2.19703 ± 0.00004 μsec. In this experiment, you will measure the lifetimes of muons that stop in a block of scintillator material, which is doped polystyrene (CH). In addition to decaying, the negative muons have a small probability of being captured by nuclei. When they are captured they disappear through decay modes that don't leave enough energy in the detector to be seen, and have a shorter effective lifetime than the positive muons. The negative muon lifetime depends on the stopping material. See Eckhause *et al.*, where it is shown that the μ^- lifetime in carbon is τ^- = 2.028 ± 0.002 μsec. Since equal numbers of μ^+ and μ^- stop in the detector, the effective mean life we expect to measure in this experiment is about τ = 2.11 μsec.

Exercise 2: *Why is the above-quoted "effective" mean life not the same as an "average" mean life?*

III. SUGGESTED READING

A.C. Melissinos & J. Napolitano, *Experiments in Modern Physics, 2nd Ed.* Chapter 8 discusses particle detectors. Section 9.4 discusses precisely this experiment, including some background information about cosmic rays. Chapter 10.5 is also useful, since the same statistics apply to muon decay as to nuclear decay.

P. R. Bevington and D. K. Robinson, *Data Reduction and Error Analysis for the Physical Sciences, 3rd Ed.* The summaries at the ends of Chapters 3 and 6 are always helpful. Sec. 7.4 describes exponential curve fitting, which is pertinent to this experiment.

Chapter 8 describes interesting non-linear techniques, which you will want to consider since you will use them in the analysis of this experiment.

Alan M. Thorndike, *Mesons: A Summary of Experimental Facts.* This book is old (1952), but is very readable, and provides a lot of information about cosmic rays. See Sections 1 and 2 of Ch. 5, especially pp. 103-108. This material concerns muon decay. The most useful information about cosmic rays is in Sec. 1 of Ch. 8. Note that the graph in Fig. 2(a) is improperly labeled. The vertical scale is too small by a factor of 10.

Hughes and Wu, *Muon Physics.* This is the modern compendium with all the current information on the subject covered and much more. Read in particular Vol. 2, Ch. 5, Sec. 2 on the muon decay spectrum and lifetime.

A. O. Weissenberg, *Muons.* Read Sections 1.1, 1.3, 1.5, and 1.6, excluding 1.6.1 and 1.6.2. Read as much of Sec. 3.9 as necessary to understand negative muon capture. Also, read Ch. 7, concerning cosmic ray muons.

Rossi, *Review of Modern Physics, v.* 20, p. 537 (1948). This article provides detailed information about cosmic rays. It is an old one, but for many years it was the bible of the field.

Eckhause, Filippas, Sutton, and Welsh, *Phys. Rev.*, **132**, p. 422 (1963). This article discusses effective lifetimes of negative muons in matter.

In reading any of the older literature, be aware that muons were called "mu-mesons" when they were first discovered. They turned out to be in the family of leptons, not mesons, so they are today never called mu-mesons.

IV. EXPERIMENTAL EQUIPMENT AND TECHNIQUE

A block diagram of the major parts of the apparatus is shown in Fig. 3. The heart of the set up is the detector, a 12" cube of plastic scintillator viewed by two 5" diameter photomultiplier tubes (PMT's) marked A and B, on opposite sides of the cube. Each has its own high voltage supply. The scintillator is made of doped polystyrene ($C_6H_5CH = CH_2$) which has a density of 1.032 gm/cm^3. A single muon stopping "event" is defined as a light signal from a cosmic ray muon entering the detector, followed at a later time by a second light signal from the muon decay electron.

The rest of the apparatus is used to measure the time interval between the light signals from the stopping muon and the decay electron and to store the information in the computer. A measure of the distribution of these time intervals yields the mean life of the muon. The total number of these events yields a measure of the cosmic ray flux for those muons stopping in the detector. A block diagram of the electronics is given in Fig. 4.

Vertically traveling muons have stopping ranges in the scintillator in the interval 0 to 32.0 gm/cm^2 of polystyrene. This corresponds to a kinetic energy range at incidence of 0 to 110 MeV. The maximum range for a muon traveling on a cube diagonal corresponds to 150 MeV initial kinetic energy. See Fig. 5 for the range-energy relations for various particle types, including muons, in CH_2; the curves are the same for polyethylene (lower density) and polystyrene (higher density).

Light signals are converted into electronic signals by the photomultiplier tubes. It is desirable to have high detection efficiency with low background from random PMT noise. Therefore, each light signal is identified by the coincidence of signals from tubes A and B formed in the high speed digital electronics. In this section pulses from A and B are each sent to separate discriminators where standard shaped logic pulses are formed for all input signals above threshold. The higher the high voltage to the PMT's the lower are the effective energy thresholds of the discriminators. With proper loading of 50Ω the standard output pulse height of a discriminator is 0.7 V, while the output widths of these pulses can be adjusted from 10 - 150 ns. For standard counting we typically use 20 ns wide outputs. Outputs for A and B (normally with zero relative delay) are then sent to the coincidence circuit, which is set to give standard logic pulses out only when pulses from A and B are both present. The output pulse width from this circuit can also be varied. Typically 100 nsec is used. One negative output of the coincidence circuit is labeled "muon" or "start pulse" and the other is labeled "electron" or "stop pulse". Scalers are also provided for counting the rates for the discriminators and the coincidence circuit during the preliminary parts of the experiment.

The start and stop pulses are now sent to the computer. The computer is set up with front-end electronics called a time to digital converter (TDC) to measure the time interval between the muon pulse from the start signal and the decay electron pulse from the stop signal. This time measure is sent to the main PC where information is stored. The timing circuit has a high precision 50 MHz clock for sorting the time intervals into histograms with 20.00 ns time bins. A maximum of 1024 time intervals or 20.48 μsec of time information can be stored.

The computer stores the histograms. It can display them in a variety of modes: linear, semi-logarithmic, or tabular, with data summed or averaged over several channels. The histograms can also be stored to floppy disk and printed out. Subsequent programs can be written to analyze data from the disk or from hand entered data from the printout. It is recommended that you take the data to a more modern computer for analysis using Excel or other software.

Three points of detail should be noted here:

1) The start pulse has provision for introducing a time delay. Since the timing circuit in the PC does not measure negative times between start and stop, putting delay here effectively cuts off time intervals of short duration. This prevents an excessive build up of events at early time due to the start and stop pulses arising from the same scintillation pulse. The latter can occur for large pulses whose tails cause the discriminators to refire. A delay of > 100 nsec here is reasonable.

2) The timing circuit is inefficient for very short times. It is a good idea to take some test data for a short time with the time spectrum spread over the screen (*e.g.* bins 1 to 60) in order to check where the circuitry is fully efficient.

3) There are several ways that fake or background events can be generated. Noise from both phototubes can make chance coincidences. Most muons traverse the entire detector and do not stop in it. There is low level radioactive background registered by the detector. There are also other possibilities. All these can give proper coincidences of A and B. If two such coincidences follow one another within 20 μsec it will give rise to a background event. (Although there is a very slight variation with time, the background rate is essentially <u>independent</u> of time.) Nevertheless, the experiment is clean enough that such background rates are usually small compared to the proper effect observed.

Exercise 3: *Does the energy absorption in the scintillator in this experiment occur all in one spot, or is it distributed throughout the volume, or in some other way?*

Exercise 4: *Read about how a PMT and base work, and be prepared to explain your understanding to your instructor.*

Exercise 5: *Explain the difference between a histogram (such as you will be recording in this experiment), and a graph that plots a given function y(x).*

V. EXPERIMENTAL PROCEDURE

NOTE: All cables used should have 50Ω characteristic impedance. They should be terminated at both receiving and sending ends either by the circuitry or externally. The PMT's are already terminated at the sending ends. The discriminators have internal termination at the input. Their outputs have provision for external termination. The coincidence circuit has internal termination at the inputs. *Its outputs to the computer should not be terminated at the sending end because it makes the signal level too small.* The oscilloscope and scalers have high impedance inputs. All circuits take negative inputs except for the scope which takes either sign. Also note that the high voltage, HV, to the PMT's must be <u>negative</u> and < 2800 V!

A. Preliminaries - not to be included in final paper.

In this part of the experiment the student will use the scope to study pulses generated by the photomultiplier tubes as they proceed through the electronic system. The student will also make use of counting rates on digital scalers.

1) Raw pulses from PMT A.

Connect PMT A directly to the scope. Turn on the high voltage gradually to 1900V. Observe and characterize the largest pulses (these are due to cosmic ray muons and decay electrons). Note their pulse heights, their rise and decay times and the fact that they are smooth. Do the same for the smallest pulses. These are single electron noise pulses and have reasonably standard shapes (the ringing is an artifact of the electronics). In between are small scintillation pulses with irregular wiggles. Can you guess why?

Now look at pulses with a ^{60}Co source on the detector. Since these pulses correspond to a maximum of ~ 1.0 MeV this offers an extremely crude way of calibrating the energy scale (It's the only one we have). The largest pulses from muons should correspond to 100 - 150 MeV. Does this check out? Keep the source away from the detector for the rest of the experiment.

2) Discriminator pulses from A.

Set the attenuator at 0 db. Check the pulse height and the rise and fall times of the discriminator outputs. Vary the width setting and note the full range that is possible. Set up to observe the input to the discriminator while triggering the scope on its output. When through studying the discriminator, set the output width at 20 ns. At this point check out PMT B and its discriminator, as rapidly as possible. Just make sure that it is operating like channel A.

3) The coincidence circuit.

Plug the discriminator outputs for A and B which normally go into the coincidence unit into the oscilloscope. Observe the behavior of the signal from B when triggering the scope on A and vice-versa. Verify that the relative timing of the pulses is such that the coincidence circuit will register all coincident pulses. Then plug the discriminator signals back into the coincidence circuit. With the coincidence circuit set for doubles and with A and B as input, check the output on the scope. Note the pulse height, the rise and fall times and the range of output widths possible. Set the width at 100 nsec and do not back-terminate. Feed one of the outputs to one of the visual scalers as a way to count the total flux of particles and background in the scintillator.

4) High voltage plateaux.

The purpose of this part is to find the settings of HVA and HVB which will yield high efficiency for scintillation pulses without getting swamped by chance noise coincidences.

Set HVB at 1800V and count coincidences on a scaler for $\geq$ 30 sec for each of several different settings of HVA. Make a plot of the data. Repeat this for HVB = 2000, 2200, and 2400V. Note that there is a good plateau region for lower values of HVB, but that it washes out for higher values. That's because at higher voltages the chance noise coincidences take over from the coincidences from real light pulses. Most often plateaus are done with independent counters so that the proper high voltage settings for the counters are independent of one another. In our case, however, both tubes look the same light pulses. Therefore, the high voltage values are highly correlated. For example, lower voltages on one tube requires a large light pulse to be present which in turn allows for a lower voltage on the other tube. It may appear that for high efficiency HVA and HVB should be $\geq$ 2200V. However, the data will not be very clean at this high setting. There will be more random background and more spurious coincidences. So it is best to back off to HVA = HVB ~ 2100 V to take reasonably clean data with, at the same time, reasonably high efficiency.

5) Watching decays on the scope.

It is possible to observe μ-e decays on the scope simply by observing an output of the coincidence circuit. Set the sweep speed to 20 μsec full scale; this is the same range as what the computer records. Set the digital scope for "infinite persistence" so that all the traces remain in the screen. You should be able to see ~ 5 -10 decay electrons/min within the first 2-4 μsec and an occasional background signal at later times.

6) Calibrating the TDC.

The interval in each bin of the time-to-digital (TDC) converter in the computer is supposed to be 20.00 nsec. If suitable delay modules are available, determine the degree to which this is the case.

7) Final check.

As a final check on the whole set up take data with the full system for 30-60 min. You can check with the computer set either to linear or semi-log display that the lifetime curve has a reasonable shape, that the background level is adequately low, and that the mean life is within 10% to 20% of the expected world average value. You will want to see the full time display from 1 - 1025 time bins. If you're not satisfied, adjust the apparatus and check again.

B. Data Taking

1) Mean life measurement. If you are satisfied with running conditions, keep running for ≥ 50,000 muon decay events. This will take > 3 days, so you may have to make arrangements with the other section to run over a weekend for final data. Remember to clock the total live time of the apparatus for your run. If you need to reduce the background, lower the HV on each tube by similar amounts such as 50 to 100 volts or more, and restart. When finished with the run save your data on a disk.

2) For the measurement of the flux of stopping muons you may choose to use the same data as for the mean life, or it may be appropriate to raise the HV to get higher efficiency. If a special run is needed it need not be long, either <1 lab period or overnight, since the approximations we use in calculating the flux make this a less precise measurement anyway.

VI. DATA ANALYSIS FOR MEAN LIFE

The radioactive decay law, written to apply to the data of an experimental run, is the following:

$$N(\mathrm{t}) = N_o\,\mathrm{e}^{-\lambda t} + \mathrm{B} = N_o\,\mathrm{e}^{-t/\tau} + \mathrm{B} \qquad (2)$$

N(t) is the number of counts in one time interval of the histogram at time t, N_o is the number of counts in the time interval at t=0, λ is the decay probability per unit time, τ is the mean life, and B is the background rate (assumed to be time-independent), with $\tau = 1/\lambda$. Thus $N(t) = (dN/dt)\,\Delta t$, where Δt is the width of a time bin. When plotted on semi-log paper the first term yields a straight line with slope equal to $-\lambda$ and intercept N_o. When the whole equation is plotted the exponential term dominates at small t and the background rate, B, dominates at large t. The total number of muons detected is then

$$N_{TOT} = \frac{N_0}{\Delta t}\int_0^\infty e^{-t/\tau}dt = N_0\,\frac{\tau}{\Delta t} \qquad (3)$$

There are several ways of displaying the data and analyzing it. The "eyeball" scheme should be used, as well as the mathematical least- squares techniques.

1) Plot the raw mean life data by hand or using Excel. Have proper error bars displayed. By eye, estimate the time at which the rate has dropped to about 40% of its maximum. Estimate your uncertainty in your “eyeball fit”. This is your first result.

2) Using proper mathematical least-squares fit to the data using Eq (2), find the muon mean life and its error, and find the total number of muons and its error. Does the value of χ^2 indicate a good fit? Is there adequate agreement with the results of the eyeball fit?

3) (Re-) bin the data in time interval bins of 20, 100, and 200 nanoseconds. This can be done conveniently in Excel, for example. (See Appendix A.) Apply your least-squares fitting technique to each binning of the data. Carefully compare your results: is the measured lifetime dependent on the binning of the data?

In your least-squares fits you have at least three parameters: N_0, τ, and B. Pick one of them and plot χ^2 vs. your selected parameter. The best fit value of the parameter will be the one which gives the best value of χ^2. How does the value of χ^2 vary with parameter value? The error in the parameter is obtained from the points at which $\chi^2 = \chi^2_{min} + 1$; this is true as long as the parameters are uncorrelated.

Exercise 6*: Derive the relationship between the half life of the material, $T_{1/2}$, and the mean life, τ, which is defined as $\tau = 1/\lambda$.*

VII. DETERMINING VERTICAL MUON FLUX AND TOTAL PARTICLE FLUX

There are two fluxes we can measure. First, there is the total flux of *all* particles (not just muons) passing through the detector, not caring whether or not the particles stop. Second, there is the flux of *stopping* muons, a number which is very much smaller.

The *total* particle flux is defined as the number of particles per second per unit area of the detector. Supposing that all coincident counts in the experiment are from real cosmic rays and not local radioactive background, you can easily estimate this number and compare it to the accepted value found in the *Review of Particle Properties* handbook.

For the stopping muon flux, use the results of the least squares fit to get the total number of muon decay events counted, $N_{TOT.}$ Recall that the flux of particles varies from the zenith angle as $F_o\cos^2\theta$, where we introduce F_o as the flux at zero degrees (from straight up). We can write a general definition of the stopping muon flux, which is a function of zenith angle, muon momentum, muon range, and the thickness of the detector:

$$F(\theta) = \frac{Muons\ at\ zenith\ angle\ \theta}{\left[Horizontal\ \ Area\ \Delta A\right]\left[Particle\ Range\right]\left[Solid\ \ Angle\ \Delta\Omega\right]\left[Time\ \Delta t\right]} \qquad (4)$$

In order to approximate the average flux measured in this experiment for those muons stopping in the detector, we make a number of simplifying but reasonable assumptions and approximations. The calculation becomes relatively straightforward and should yield an average value for the flux which is reliable to within 20%.

Though there are corrections to be made later for inefficiencies, we start with the following simplifying assumptions:

1) Empirically, $F(\theta) = F_o \cos^2 \theta$ where F_o is the flux at zenith.

2) We assume that the number of stopping muons per unit volume in the scintillator is independent of position in the block of polystyrene. In other words, the stopping power of CH_2 is independent of range, and the spectral distribution for these low energy stopping muons is also independent of range. This is an rough approximation, but adequate for the accuracy of the result we are aiming for.

3) The efficiency for detection of μ-e events is 100%.

Now finding the vertical flux F_o from the total number of stopped muons detected is straightforward. First, relate it to the total number of stopped muons detected:

$$N_{TOT} = \int F_o \cos^2\theta \, dA \ \ d(Range)\, d\Omega \, dt \qquad (5)$$

where the solid angle element is written as

$$d\Omega = \sin\theta d\theta d\varphi \qquad (6)$$

which, after integration, leads to

$$N_{TOT} = F_o \frac{2\pi}{3} \rho V\, T \qquad (7)$$

where

N_{TOT} = the total number of decay events counted in a run, as defined previously;
T = the total time for counting;
$\rho V = \int dA\ d(Range)$
ρ = the density of polystyrene, and is equal to 1.032 gm/cm^3.
V = the volume of the detector.

Corrections are made for a number of reasons. The first is straightforward and must be made in any event. You should also correct for the second. The others would require further research and thought. Comment on the size of their effect in your report.

1) If the start pulse was delayed with respect to the stop pulse, efficiency was lost. In effect, time starts at some time t_{Delay} instead of t=0. You know the delay time so you can compute the correction to the total number of counts.
2) The discriminators have finite thresholds causing reduced efficiency. Thus, some muons that barely enter the scintillator block before stopping, and some electrons that leak out, may not trigger the discriminators.

3) The light collection efficiency for the scintillator block is position dependent. Less light will get to the phototubes from a corner than from the center.

4) The range-energy relation is not uniform. See Fig. 5.

5) The flux/unit range is not constant over all stopping muon ranges. See Fig. 1.

The accepted value for the flux of *stopping* cosmic ray muons is:
5 x 10^{-6} muons per sec per cm^2 per steradian per (gm/cm^2) of material. The uncertainty on this number is unknown. Thus, you need to be cautious when comparing your result (with estimated uncertainty) to this number.

VIII. RESULTS AND CONCLUSIONS

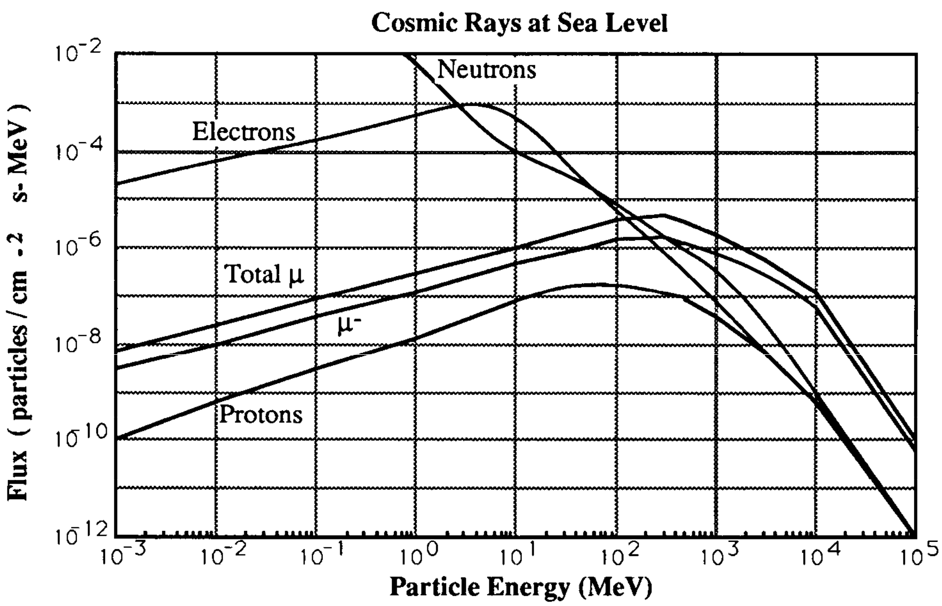
As with all other experiments, draw conclusions from your results and make a comparison of them to accepted values. That is, do your results agree with accepted values? Compare your lifetime to the one given at the beginning of this handout. Compare your flux values to the ones given above. Typically students get lower flux measurements the accepted values. Any ideas as to why? Investigate possible sources of systematic errors. How large are they?

Specifically, in your article your goal is to give results for:

1) The lifetime of cosmic muons stopped in material.

2) The total flux of all particles going through the detector.

3) The vertical flux of stopping muons in the detector.



Flux of cosmic ray particles at sea level at 40° N geomagnetic latitude. Data from J. Ziegler, Nucl. Instr. Methods, 191 (1981) 419. Below 3 MeV for electrons and about 10 MeV for protons the fluxes depend on local atmospheric conditions.

Figure 1: The flux of cosmic rays particles at sea level, including muons, as a function of particle kinetic energy. The vertical axis is in units of particles per square centimeter per second per MeV interval of kinetic energy, *i.e.*, $\frac{particles}{cm^2 \sec MeV}$.

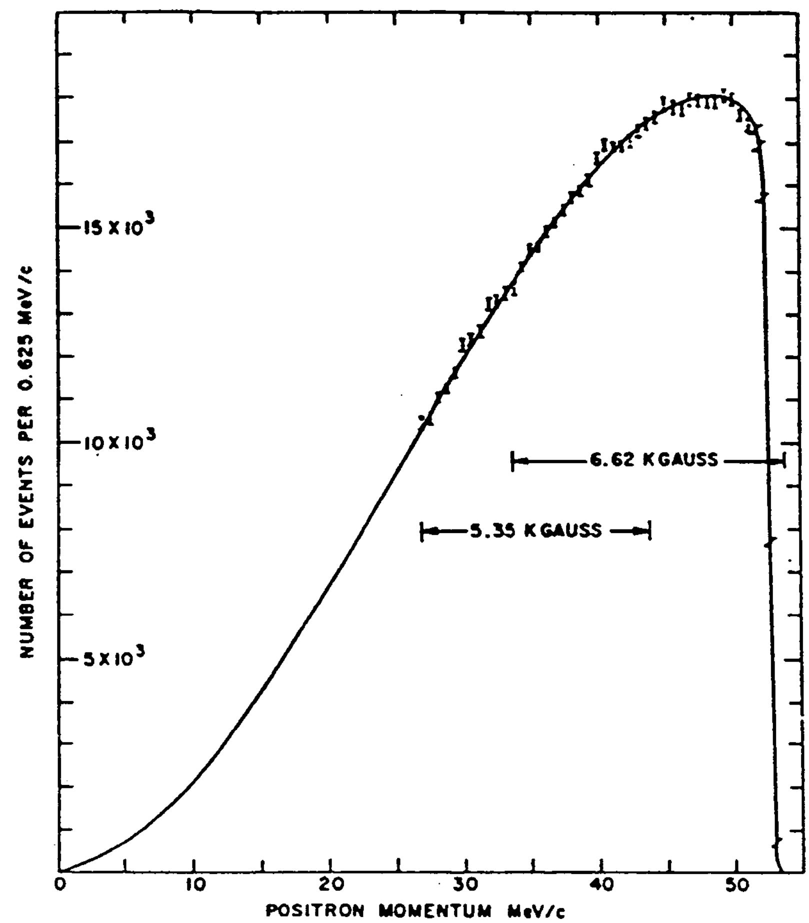


Fig. 3. Data from the experiment of Bardon *et al.* (1965). The experimental points are compared to the theoretical curve for $\rho = \frac{3}{4}$, modified for radiative corrections and experimental resolution.

Figure 2: The distribution of electron kinetic energies in muon decay at rest.

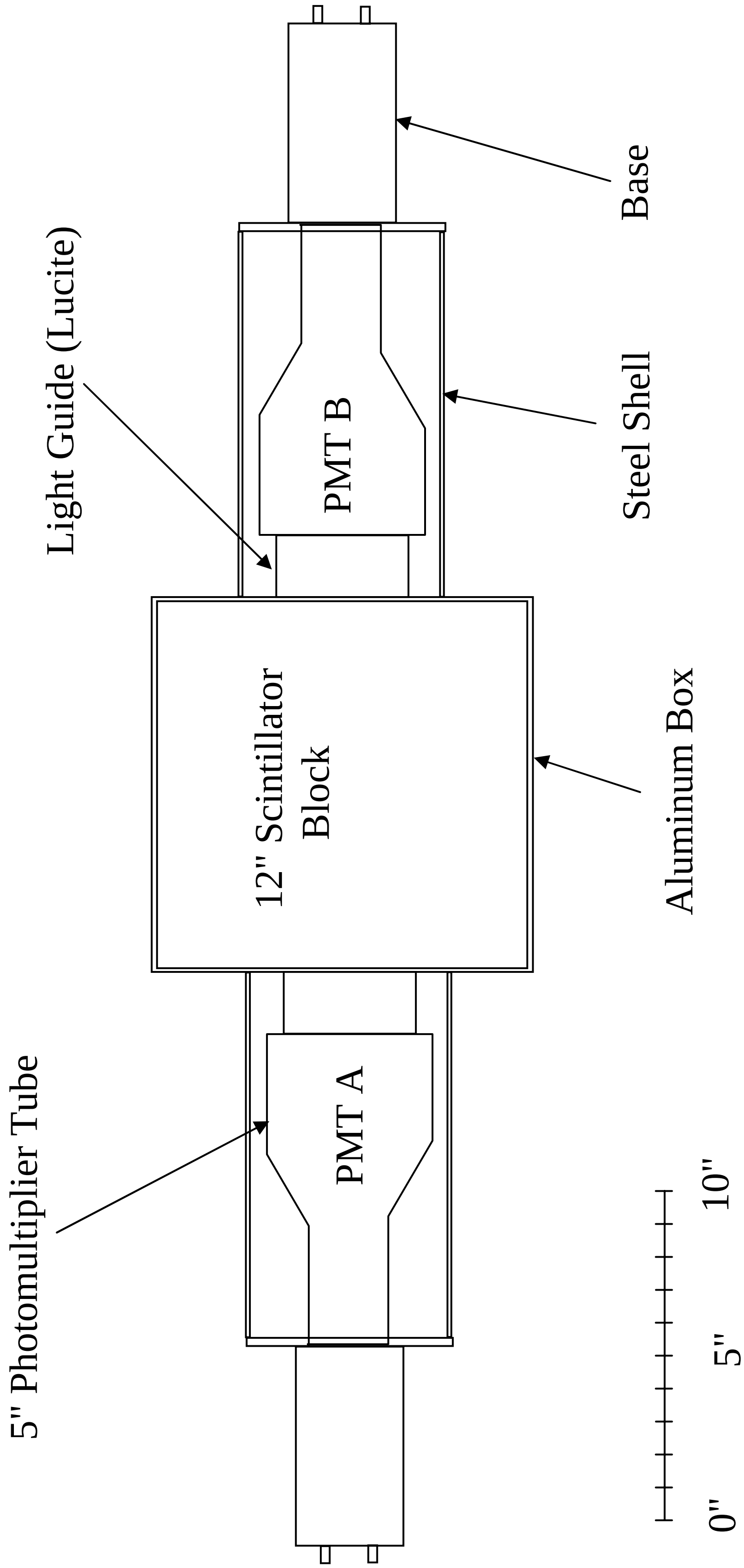


Figure 3: Detector setup, approximately to scale.

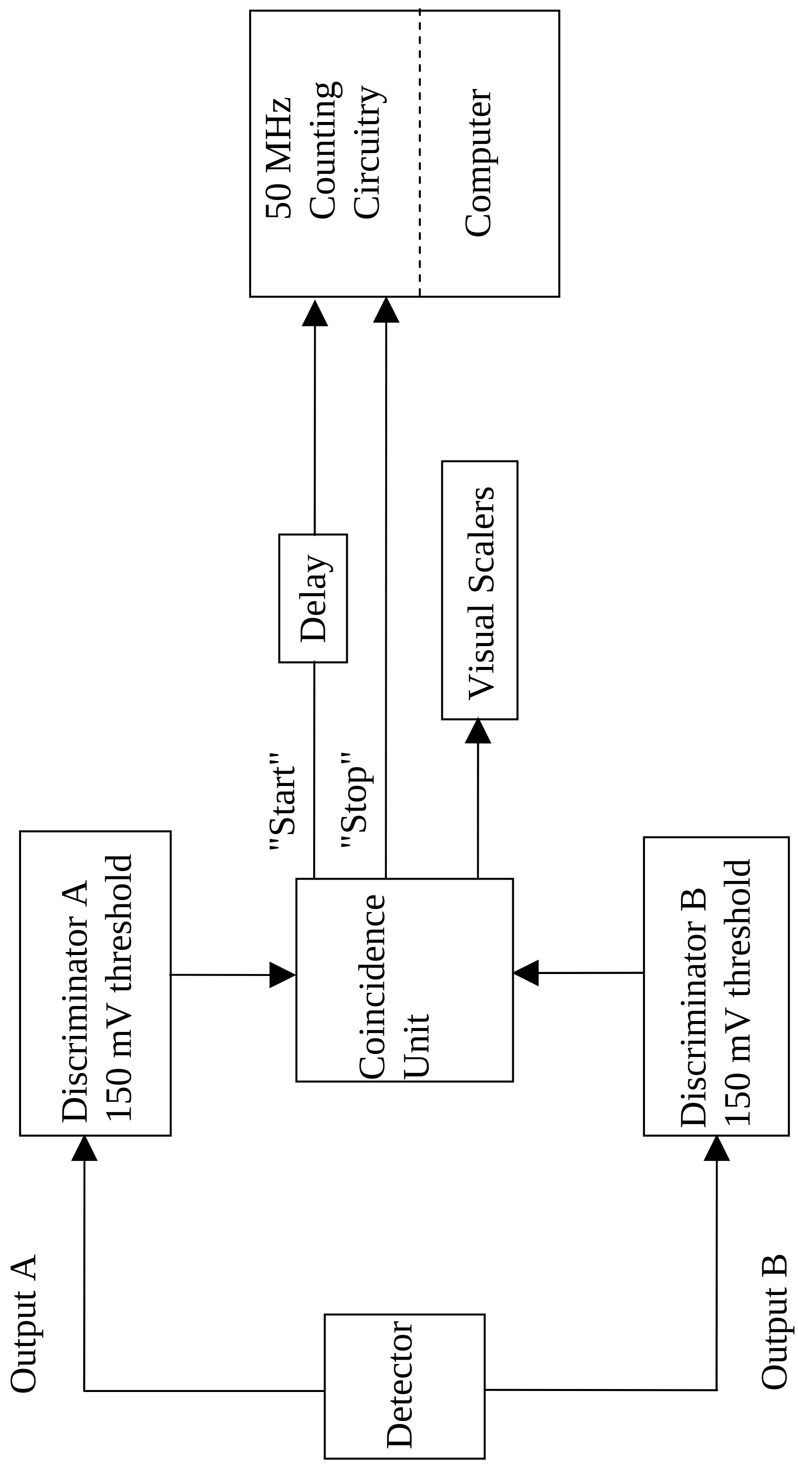


Figure 4: Block diagram of event counting circuit.

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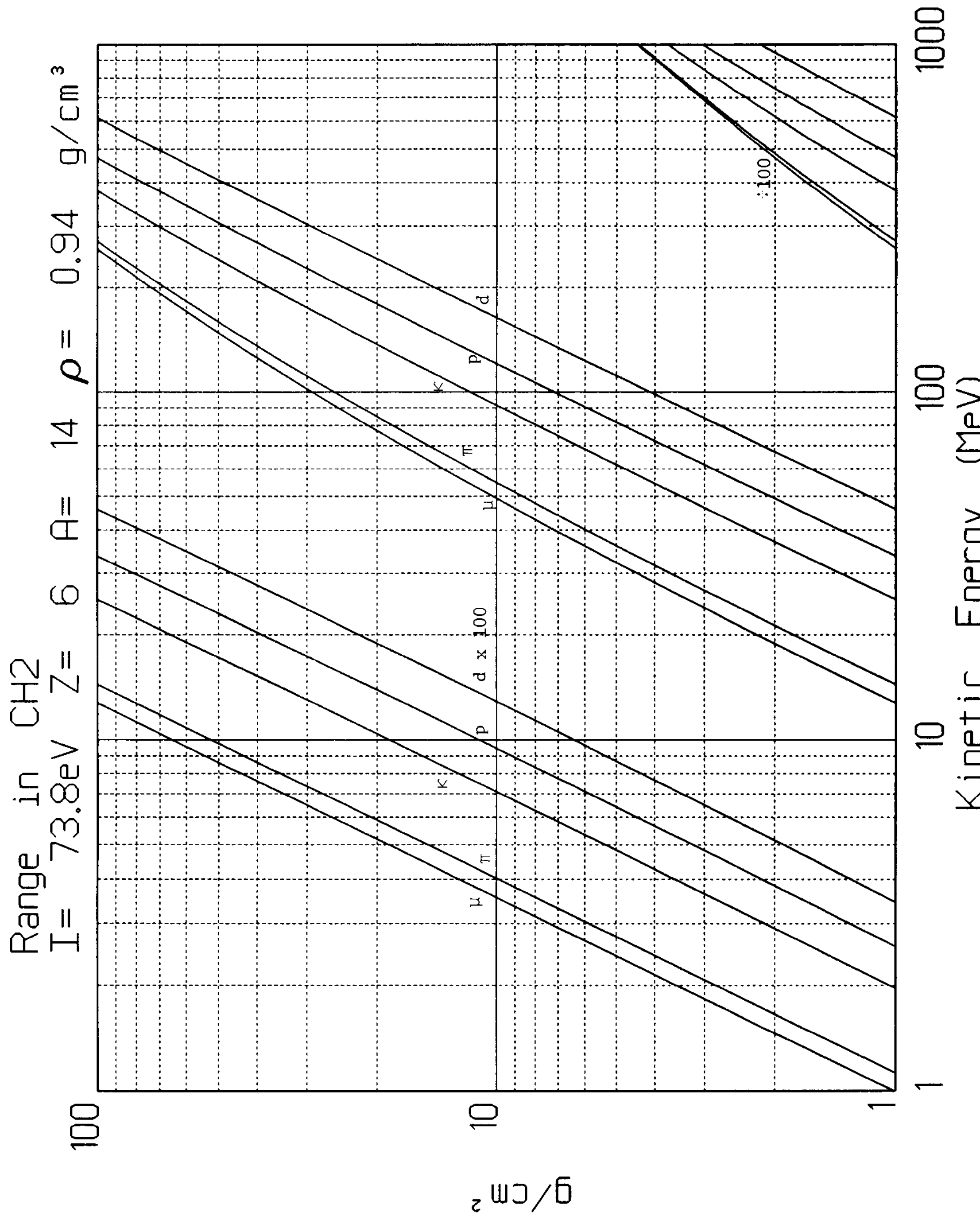

Figure 5: The range of various particle types, including muons, in polystyrene or polyethylene (both CH_2) as a function of the particle kinetic energy in MeV. The range is given in units of grams per square centimeter, which is the thickness in cm times the density of the material in gm/cm^3.

Appendix A: Rebinning the data.

Converting the data from the initial 20 nsec binning to coarser bins is done to improve the counting statistics in the low-count bins. However, in Excel it is not obvious how to take a list of 1024 bins of numbers and sum them in groups of 5 or 10 bins to make a new set of numbers in coarser bins. Student groups have found numerous tricks to make this work, some easy, some complex. Here is one method that requires no real effort:

1) Start with the list of 1024 numbers in column A and sum groups of, say, 5 numbers by setting each 5th cell in column B to "=sum(A1:A5)" , etc. Thus, column B will have a number followed by 4 blank cells.
2) Select column B and press the F5 button which brings up a window titled "Go To", wherein you select "Special..." In the new window that opens, select "Blanks". This will highlight all the blank cells.
3) From the Edit menu select "Delete". Pick the option "Move cells up" when asked. This gets rid of the blank rows.
4) Cut and paste the newly-condensed rebinned column, as needed, to display the results.