# COMPTON SCATTERING OF GAMMA RAYS

v2.0
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## I. INTRODUCTION

Compton scattering is the name given to the scattering of high-energy gamma rays from electrons. The gamma rays come from the decay of certain radioactive nuclei, and the electrons are ordinary atomic electrons found in any atom upon which the gamma rays impinge. The importance of this phenomenon is that it shows that gamma ray photons can be treated "like particles" in as much as they obey the same momentum and energy conservation laws as ordinary particles when they scatter from the electrons. Your goal in this experiment is to verify that the energy of the scattered photons (gamma rays) is given by the formula derived from Special Relativity (Eq. 6.3.6 in Melissinos), and to see whether the cross section for scattering as a function of angle better obeys the Klein-Nishina equation (Eq. 6.3.11) or the Thomson equation (Eq. 6.3.9).

BEFORE conducting the experiments in this write-up, consult your instructor about laboratory *radiation safety* procedures.

## II. BACKGROUND READING

The following references will provide useful background for the experiment and should be perused:

| Reference | Sections | Topics |
|---|---|---|
| Melissinos[1] | 9.1 – 9.2 | Compton scattering |
| | 8.4 | Scintillators |
| | Appendix D | Radiation safety |
| | Chapter 8 | Particle detectors |
| Eisberg & Resnick[2] | 16.3, 16.5 | Beta and gamma decays |
| Bevington & Robinson[3] | 2.2, 2.3 | Poisson and Gaussian distributions |
| | 3.2, 3.3 | Error propagation |
| | 6.1-6.6 | Least-square fitting for a line |
| | 11.1 | Goodness of fit test |
| Firestone *et al.*[4] | | Table of Isotopes, 8$^{th}$ Ed. |

## III. EQUIPMENT

The apparatus is similar to that shown in Fig. 6.10 of Melissinos. Not shown in that diagram is the computer used for recording the data. Incident gamma radiation of 0.662 MeV energy is provided by a $Cs^{137}$ source of about ½ millicurie. The cesium source is housed in a lead box, and is located approximately 14" from the aperture at the front. The diameter of the aperture is 1", and the source position is marked on the exterior of the lead box. This defines a "beam" of gamma rays from the source which diverges predictably in angle.

The NaI(Tl) crystal we use to detect the gamma rays is 3" diameter and 3" deep. It is coupled optically to the photomultiplier tube (PMT), at the base of which is the PMT voltage divider and preamplifier. High voltage is brought into the base from the HV supply. Pulses are fed into the input in the back of the computer which serves as a multichannel pulse-height analyzer. In front of the detector you can place a lead collimator with a diameter smaller than the diameter of the detector. This is used to define the "solid angle" into which gamma-rays scattered from the target will hit the detector.

The distance of the detector from the target should be such that all straight-line rays from the source which pass through the aperture in the box should impinge upon the surface of the detector. Typically, the target-to-detector distance is 8" to 10". You will need to carefully measure both this distance and the area of the detector to compute its solid angle.

For larger scattering angles the count rate may be very small. This problem can be solved by running over a very long period of time (overnight, for example). Alternatively, a larger collimator or shorter detector-target distance can be used to increase the scattered flux.

The data acquisition computer takes the analog signals from the PMT base and integrates the amount of charge in each pulse in a circuit called an analog to digital converter (ADC). The size of the pulse is proportional to the amount of energy deposited in the detector by each "event". The pulse-height analyzer then accumulates this information in the form of a histogram in which each channel corresponds to a definite (but initially unknown) energy, and the number of events in each channel reflects the number of times a given amount of energy was deposited. The resultant histogram is referred to as a "spectrum".

***Exercise 1****: Explain whether the energy absorption in the scintillator in this experiment occurs all in one spot or whether it is distributed.*

***Exercise 2:*** *Explain the difference between a histogram (such as you will be recording in this experiment), and a graph that plots a given function y(x).*

***Exercise 3:*** *Read about how a PMT and base work, and be prepared to explain your understanding to your instructor.*

## IV. ENERGY CALIBRATION PROCEDURE

Prior to measuring Compton scattering you will need to calibrate the energy scale of the system. This will be done with a set of known sources. The following steps are recommended. This part of the experiment may take most of your first lab session.

1) Set the detector near $90^o$ and keep the main source closed with a lead brick. Put a $Cs^{137}$ source near the front of the counter; best is to put the source at the "pivot point" where the scattering target will sit later, but you get a much higher countrate if you put the source right in front of the NaI. Turn on the HV and bring it up to 1400 Volts. **(Keep the high voltage below 1500 Volts at all times.)** While doing this, view the output pulses on an oscilloscope. These will be seen to be negative pulses, fast rising and slow (50-200 nsec) falling. Observe the height of these pulses as you lower the HV on the PM. Bring HV back to 1400V.

2) Next, feed the output of the PMT to the computer. Using the $Co^{60}$ source, set the gain on the computer until the highest energy peak falls toward the right-hand side of the display. Initially the computer displays "channels" or "bins" on the horizontal axis. Eventually you want a horizontal scale calibrated in either keV or MeV.

3) Return to the $Cs^{137}$ source and accumulate a spectrum (similar to Fig. 5.28 of Melissinos) for a minute or so and observe the following features:

i. the photo peak and channel number around which peak is centered;
ii. the full width at half maximum (FWHM) of the photo-peak;
iii. the Compton edge and continuum.

Is the distance in channels between the photo-peak and Compton edge roughly what you expect? Explain.

***Exercise 4:*** *What is the so-called Compton edge? Calculate its expected position.*

4) Locate the center of the 662 keV peak and record the channel number, $n_\gamma$. Also determine the full width at half maximum (FWHM), $\Gamma_{FWHM}$, for this peak expressed as a number of channels. Repeat using $Na^{22}$, $Co^{60}$, and $Sn^{113}$ sources. The computer could now do a "two-point" or "three-point" energy calibration for you, but it cannot evaluate the goodness of the calibration. Therefore, plot the peak channel numbers versus $\gamma$ energy yourself and confirm the linearity of the detector/counter system in response to the energy of the incident $\gamma$-rays. Do linear least-squares fit using a function of the form

$$E_\gamma = a + b\, n_\gamma \qquad (1)$$

and determine the values and associated errors of the coefficients a, and b. State what the expected error would be in determining the energy of an unknown photon line in the calibrated region if its associated bin number is well known.

5) Estimate the full width at half maximum ($\Gamma_{FWHM}$) of each significant peak, in keV. (This will be difficult for some peaks due to large background corrections.) Make a plot of $\Gamma_{FWHM}^2$ versus $E_\gamma$ and perform a linear least-squares fit with the function

$$\Gamma_{FWHM}^2 = c\,E_\gamma + d \qquad (2)$$

***Exercise 5:*** *Figure out why this functional dependence may be applicable.*

The known gamma-ray sources you have available are

| | |
|---|---|
| $Sn^{113}$ | (0.392 MeV) |
| $Na^{22}$ | (0.511, 1.274 MeV) |
| $Cs^{137}$ | (0.662 MeV) |
| $Co^{60}$ | (1.17, 1.33 MeV) |

6) The detector's calibration may vary with angle or source location and affect your results. At some point you should pick some well-defined calibration source line (one for which data accumulates fast) and measure its position as a function of detector angle and source placement. If it varies significantly you'll need to repeat the energy calibration at several angle settings. Discuss what you think "significant" means with your instructor.

The results of your energy calibrations should NOT be part of your journal article. At most, the reader may want to see one of them as a sample. Nevertheless, they should all be in your laboratory notebook.

***Exercise 6:*** *How are the half-life and the mean life of a radioactive substance related?*

## V. COMPTON SCATTERING

There are two goals. The first is to measure the energy of the scattered photons as a function of angle and to determine the mass of the scattering centers (i.e. the electron mass). The second goal is to measure the scattering cross section as a function of angle and compare it to two "competing" theoretical models in order to pick the better one. The data you accumulate can serve both purposes at once, but you may find it easier to get started if you pursue the energy dependence first and then go back to measure the cross section.

*1) Gathering data*

You should measure the scattering at about 10 angles between 50° and 130° inclusive. At each angle you run for about 5 minutes with the beam blocker removed but with no scatterer in place. If you see no prominent peaks in the upper part of the spectrum then you can be confident that the background radiation will not mask your true signal. Put the ½" aluminum plate (our supply of scattering electrons) at the target position. It should be oriented so that the normal to its plane is at one-half of the scattering angle. It makes a difference whether the Compton-scattered gamma rays go through the whole target (“transmission mode”) or whether they come our of the same surface they went in (“reflection mode’). (Figure out why.) Run for as long as necessary to get a well-identified peak from the scattered gamma rays. Save the spectrum in the computer. Typical γ spectra of $Cs^{137}$ obtained when the counter sees the source directly and when it looks at scattered photons are shown in Fig. 6.11 of Melissinos.

For each angle setting you need to extract the number of counts in the peak. This involves subtracting any background upon which the peak sits. Discuss this with your instructor. You also will need the energy corresponding to the centroid of the peak. For this you need an energy calibration at the given angle, as discussed above. Also record the width of the peak. The width will be greater than that of a mono-energetic γ of the same energy. How much greater is it and what are the sources of broadening?

You must be certain, at the smaller angles, that the open area of the counter cannot see the edge of the aperture in the source housing; if it can see it, you will record scattering from the collimator lip. Below about 50° this is harder to do, so that small angles cannot be used with our set-up. At the smaller angles, you can use the ½” diameter collimator in front of the source. Start your data taking at some intermediate angle, such as 60°.

*2) Energy analysis*

If the energy of the incoming gamma ray is *E* and the energy of the Compton-scattered gamma ray at angle θ is *E´*, and if the mass of the scattering center is *m*, then Special Relativity says, using energy and momentum conservation, that

$$E' = \frac{E}{1 + (E/mc^2)(1-\cos\theta)} \qquad (3)$$

Reduce the energy data and plot it as in Fig. 6.13 of Melissinos. Calculate the electron mass and the primary gamma energy from the curve. In all data reduction and presentation (graphs), include the best error estimates you can make. If some of your results deviate from what you expect, find the reason why and repeat where practical. Remember that the calibration of the detector is energy dependent and that you must account for this effect to get the best results.

***Exercise 7:*** *Derive the energy equation from first principles.*

*3) Cross section analysis*

This part of the experiment is to measure the scattering cross section as a function of energy and compare your results with the Klein-Nishina formula (Eq. 9.14, Chapter 9 of Melissinos) and the Thomson formula (Eq. 9.11 in Chapter 9). The former is the full quantum-mechanical result using photons, while the latter is the classical result built on wave-like scattering.

Calculate the differential scattering cross-section at each of your selected scattering angles. It is a measure of the probability that gamma rays scatter at a given angle, per scatterer, but it is expressed in terms of an effective area; your instructor can explain this in detail. The formula is derived in most books of classical mechanics in the context of celestial mechanics or nuclear scattering. A new concept to become acquainted with is the notion of a “solid angle”, Ω, which is a measure of an angle in three dimensions; roughly speaking, it is a cone angle, and it has units of radians$^2$ that are called “steradians”. The formula for cross section can be written:

$$\frac{d\sigma}{d\Omega} = \frac{F_{out}}{F_{inc}\Delta\Omega \ Z\,(\rho L(N_o / A))} \frac{1}{\eta} \frac{\varepsilon_{inc}}{\varepsilon_{out}} \quad (4)$$

where

- $F_{out}$ = measured number of counts in scattered peak per sec
- $F_{inc}$ = incident beam flux (gamma's per second)
- $\Delta\Omega$ = geometrical solid angle subtended at the target by the detector
- Z,A = atomic number and atomic weight (grams/mole) of target
- $\rho$ = density of target in g/cm$^3$
- t = target thickness in cm
- L = target thickness presented to beam (= t/cos(θ/2) in “transmission” mode)
- $N_o$ = Avogadro's number

There are three significant corrections that have to do with the behavior of the detector and the target:

- $\varepsilon_{inc}$ = detector efficiency for γ's at the energy of the incident beam
- $\varepsilon_{out}$ = detector efficiency for γ's at the energy of the scattered peak
- η = factor to correct for photon absorption by the target

The factor in parentheses in the denominator is the number of atoms per square centimeter in the target as "seen" by the incoming beam. Be sure to specify the numerical values of all these quantities in your final journal article.

The flux of scattered gamma rays that enter your detector, $F_{out}$, is determined by integrating the number of counts in each recorded spectrum you obtain, divided by the time duration of the run. One way to do this is to use the integrated net counts estimated by the data collection software. A more precise method is to fit the measured spectrum to a Gaussian function with a suitable polynomial background. Consult with your instructor about this. Also, you can measure the flux for angles less than 50 degrees if you subtract a suitable “background” spectrum obtained with the aluminum target plate removed.

You have to know the flux gamma ray flux incident from the source onto the target, $F_{inc}$. This can be done by setting the counter at 0°, removing the lead collimator from the counter, and recording the spectrum for five minutes. Be sure to use the "live-time" clock, rather than the real time. This will measure the beam intensity only if the entire beam hits the counter. Thus, you should compare the spectra for a given time as you move the counter towards the aperture in the shielding house. When the rate is independent of distance, the counter is intercepting the full beam. Once this is done, move the counter back on the arm as far as possible. The measured counts must be corrected by the counter efficiency at 662 keV.

The incoming and outgoing measured particle fluxes $F_{inc}$ and $F_{out}$, must be corrected for detector deadtime, which is a factor given by the ratio of the real elapsed time for a measurement to the time the computer was ready to process events. Both factors must also be corrected for the efficiency of the detector for "seeing" photons of the relevant energy. This is discussed below.

The solid angle, $\Delta\Omega$, represents the portion of the full $4\pi$ range of angles in three dimensions which the detector subtends as seen from the target location. To a good approximation we can write

$$\Delta\Omega = \frac{A}{r^2} \tag{5}$$

where A is area through which particles enter the detector and r is the distance from the target to the detector. Be careful in evaluating the experimental uncertainty on this quantity.

Some of the more intricate corrections to your results are discussed in the following section. However, even with crude versions of those corrections you should plot your results and compare to theory.

Your results should be plotted as is done in Fig. 9.8 Melissinos. Plot the Klein-Nishina and the Thomson cross-sections together with your data, but plot the theories as solid curves extending over the full range of angles. Reach a definite conclusion about which theory is better supported by your data.

## VI. EFFICIENCY CALIBRATIONS: $\varepsilon$

When photons interact with matter they either deposit all their energy or none of it. There is a good chance that the photons you are trying to detect go clear through the detector without leaving any trace. We call this situation a loss of efficiency. Hence, in order to know the true $\gamma$ rate on the detector, one must compensate for the loss in detection efficiency. The photo-peak efficiency for a given counter and geometry depends on the photon energy. See, for example, Fig. 9.6 in Melissinos. You can use Melissinos's curve, but be aware that the efficiency depends on the counter size, on the aperture in front of the counter, and on the distance between source and detector. You can determine it for your particular set-up directly. There are two ways to do this: by calculation and by direct measurement.

To calculate, we know that the NaI detector is 3" thick. We can use well-known photon-attenuation information to compute the fraction of gamma-rays that will pass through the detector unseen. Figure 1 shows[5] the energy-dependent photon attenuation lengths for various materials, including NaI. For photons of initial intensity $I_o$ passing though a homogeneous material of density $\rho$ and thickness $d$, the remaining intensity, $I$, is given by

$$I = I_o e^{-\rho d/\lambda} \tag{6}$$

From this you can compute the efficiency, ε, of your detector at any photon energy of interest.

Note that if the efficiency for detecting the incoming and the outgoing fluxes were the same, then in the cross section formula the factors would cancel. However, the efficiency is not independent of energy and hence not independent of angle.

To measure the efficiency directly, we count the number of events in the photo-peak for a known incident rate. The ratio of the two is the photo-peak efficiency. Thus, you can use the known sources you used earlier to make measurements at the energies of these sources.

If you follow this route, place the source at the position the scatterer was in the scattering measurements. Make certain the correct collimator is in front of the counter; if you used more than one collimator in front of the counter, the efficiency must be measured for each. Note that the activity marked on each source was the activity at the time that the source was prepared. You will have to calculate the present activity using the known half-life and the time elapsed since the date of preparation.

One additional complication is that not all the predicted activity emerges in the form of photons you want to detect. For example, for each decaying $Na^{22}$ nucleus we get a 1.274 MeV gamma ray 100% of the time, and a positron 90% of the time. The positron annihilates with any nearby electron and produced two gamma rays of 0.511 MeV energy each. Thus, the relative fraction of 0.511 MeV to 1.274 MeV gamma rays is 1.8 to 1.0. The fact that in your data the ratio looks different is a reflection of the different efficiencies of the detector at these two energies. Also, for some of the sources you are using the outgoing photons sometimes get absorbed by the atomic electrons of the same atom from which they were emitted; this is called Internal Conversion. The half-lives, decay schemes, and various internal conversion factors may be found in the compendium called the *Table of Isotopes*.[4] Ask your instructor to orient you to the use of this book.

Compute the expected count rate for each line from the known source knowing the total rates you calculated and the known solid angle of your detector. Measure the actual count rates and compare to your predictions. The ratios are the efficiencies of the detector at the given energy.

## VII. EFFICIENCY CALIBRATIONS: η

The last correction factor, η, accounts for the fact that gamma-rays may interact in the aluminum target but not undergo Compton scattering. Some fraction of them will undergo photoabsorption in which they simply give up their energy to an electron and disappear completely. The fraction that do this depends on the thickness of the target and affects both the incoming and the outgoing gamma rays, and is also a function of energy. This is a fairly small correction, but significant for some energies and angles.

The correction proceeds along the same lines discussed with Equation (6). You can approximate that all the incoming gamma rays pass through half the target thickness on the way in, and the outgoing (lower energy) gamma rays pass through half the target material on the way out.

The relevant absorption coefficient can be found in "Table of Isotopes"[4], and on the course web site. You can also try to use Figure 1, although it does not have a curve specifically for aluminum.

## References

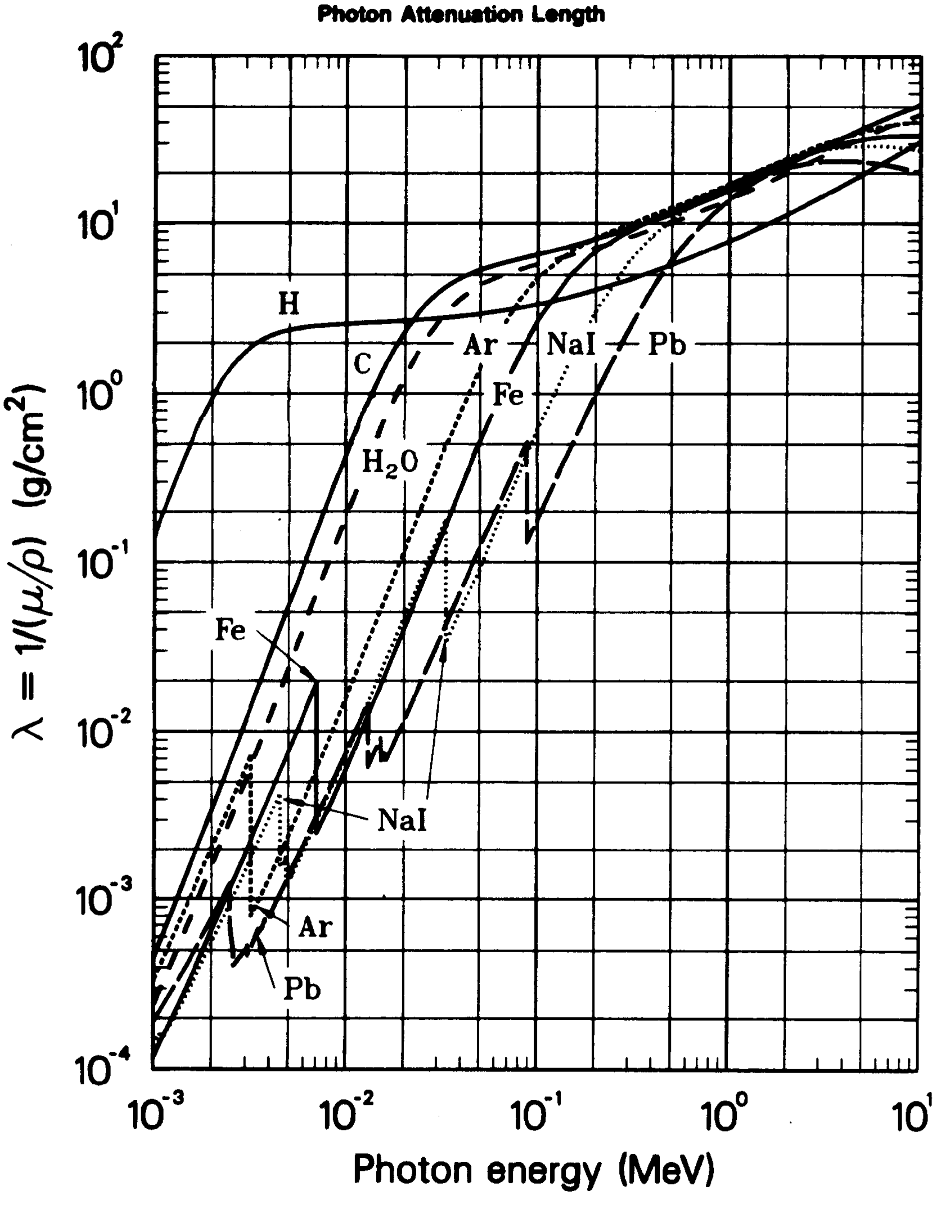
1) A.C. Melissinos and J. Napolitano, *Experiments in Modern Physics, 2nd Edition,* Academic Press, New York, 2003.

2) R. Eisberg & R. Resnick, *Quantum Physics 2nd Ed,* J. Wiley & Sons, New York, 1985.

3) P.R. Bevington and D.K. Robinson *Data Reduction and Error Analysis for the Physical Sciences, 3rd Edition,* McGraw Hill Book Company, New York, 2003.

4) R. B. Firestone, V. S. Shirley *et al*, *Table of Isotopes, 8th Edition,* John Wiley & Sons, Inc., New York, 1996. See also the web site: http://ie.lbl.gov/toi.html, which contains much of the same information, and more.

5) L. G. Greeniaus, *TRIUMF Kinematics Handbook, 2nd Edition*, 1987.




**The photon mass attenuation length λ = 1 / (μ / ρ) ( also known as the mean free path) for various absorbers as a function of photon energy, where μ is the mass attenuation coefficient.**

Figure 1: Data for computing the efficiency of the NaI detector and for estimating the loss of gamma-rays in the aluminum target. From Ref [5].