33-228 Electronics Spring 2016

**In-class Team Exercise**

**Tuesday, Week 3**

(**1**) Express the complex number $j = \sqrt{-1}$. in complex exponential form. Using this, determine $j^2$, $j^3$ and $j^4$ in complex exponential form. Using the complex exponential form, express them in Cartesian complex form, $a + jb$.

A complex number, $Z$, can be written as $|Z|(cos\phi + j\sin\phi)$, we have $Z = j$, so we must have $\phi = \pi/2$. Thus, $j = e^{j\frac{\pi}{2}}$. From this, we have

$$\begin{aligned} j &= e^{j\frac{\pi}{2}} \\ j^2 &= e^{j\pi} \\ j^3 &= e^{j\frac{3\pi}{2}} \\ j^4 &= e^{j2\pi} \end{aligned}$$

Using our expansion from above, we get

$$\begin{aligned} j &= e^{2\frac{\pi}{2}} &&= j \\ j^2 &= e^{j\pi} &&= -1 \\ j^3 &= e^{j\frac{3\pi}{2}} &&= -j \\ j^4 &= e^{j2\pi} &&= 1 \end{aligned}$$

(**2**) A time-dependent voltage is given as

$$v(t) = v_0 \cos\left(\omega t + \frac{\pi}{2}\right) .$$

(a) Express this voltage in complex exponential form.

$$\begin{aligned} v(t) &= v_0\, e^{j\left(\omega t + \frac{\pi}{2}\right)} \\ v(t) &= v_0\, e^{j\omega t}\, e^{j\frac{\pi}{2}} \end{aligned}$$

(b) What does $e^{j\frac{\pi}{2}}$ equal?

From problem 1, $e^{j\frac{\pi}{2}} = j$.

(c) Using this latter fact to simplify your complex exponential, determine an equivalent expression for $v(t)$ that doesn't involve complex numbers.

This lets us write that

$$\begin{aligned} v(t) &= j\, v_0\, e^{j\omega t} \\ v(t) &= v_0\, j\, (\cos\omega t + j \sin\omega t) \\ v(t) &= v_0\, (-\sin\omega t + j\cos\omega t) \\ v)t) &= -v_0\, \sin\omega t \end{aligned}$$

(**3**) The complex gain of some circuit can be expressed as

$$G = \frac{1}{2+j}\,.$$

(a) What is the magnitude of the gain?

The magnitude to the gain is $|G| = \frac{1}{\sqrt{5}}$.

(b) Express the gain in the Cartesian complex form, $a + jb$.

$$\begin{aligned} G &= \frac{1}{2+j} \\ G &= \frac{2-j}{5} \\ G &= \frac{2}{5} - j\frac{1}{5} \end{aligned}$$

(c) Express the gain in complex exponential form.

The phase angle is given as

$$\begin{aligned}\tan\phi &= \frac{-1/5}{2/5} \\ \tan\phi &= \frac{-1}{2} \\ \phi &= -26.56^\circ \\ \phi &= -0.4636\,\mathrm{rad}\end{aligned}$$

Thus, we get that

$$G = \frac{1}{\sqrt{5}}\,e^{-j0.4636}\,.$$

(**4**) Consider a gain function given as

$$G(f) = \frac{1}{1 - j(f/1\,kHz)}\,.$$

(a) What is the approximate gain for $f << 1\,k\,Hz$?

For frequency small compared to $1\,k\,Hz$, the gain is $G \approx 1$.

(b) What is the approximate gain for $f >> 1\,k\,Hz$?

For frequency large compared to $1\,k\,Hz$, the gain is

$$\begin{aligned}G &\approx \frac{1}{-jf/(1\,kHz)} \\ G &\approx j\frac{1\,k\,Hz}{f}\end{aligned}$$

(c) Will $|G|$ be maximal for $f < 1\,kHz$, or for $f > 1, kHz$?

When $f$ is small, the gain is 1, which is maximum.

(d) What is maximum attenuation, $20dB\log_{10}|G|$, for this filter?

The maximum gain is 1, so the attenuation is $0\,dB$.

(e) At what frequency is the attenuation $3\,dB$ below the previous value?

The $-3\,dB$ point occurs when the gain is $|G| = 1/\sqrt{2}$. This happens when $f = 1\,k\,Hz$.