Recitation Problems for Week 4, Thursday

3.S47. For a solid, uniform sphere of mass M and radius R, the gravitational force on a mass m inside the sphere is computed by using the contained mass, M_c of the sphere which is inside the radius of the mass m interacting with the mass m. For m at a radius r, we have

$$M_c = M\left(\frac{r}{R}\right)^3 .$$

(a) Show that the resulting force on the mass m is proportional to its distance from the center of the solid sphere. (b) A classic physics problem discusses drilling a well through the center of the Earth coming out on the other side of the planet. Ignoring air friction, how long would it take for an object released from the surface of the Earth to fall to the other side, and then return to where its starts?

S55. A roller coaster is traveling along a track with bumps and dips in it. We can approximate the top of the bumps and the bottom of the dips as circles of radius R. (a) How fast will the roller coaster need to travel over a bump for the rider to feel weightless? (b) How fast will the roller coaster need to travel through a dip for the rider to feel four times has heavy as they normally do?

3.S56. In a planetary mission, a spacecraft of mass m established a circular orbit of radius R around an asteroid.. If the period of the spacecrafts orbit is T, what is the mass of the asteroid, M? (b) If $m = 500\,kg$, $R = 45\,km$ and $T = 1.04$ days, what is the mass of the asteroid, M?

3.S57. In discussing the evidence for dark matter in Section 2.6, we saw that there was insufficient visible mass in galaxies to explain the speed of the stars orbiting the center of the galaxy. The observed rotation curves were inconsistent with what we expected for the mass concentrated at the core of the galaxy. For a typical galaxy, the visible mass is on the order of 10^{11} solar masses, and 90% of this is concentrated in central core of the galaxy (about 10% of the observed radius of the galaxy). One theory of dark matter explains the dark matter as MACHOs, *massive astrophysical compact halo objects*. In this picture, the MACHOs are made of normal matter, but too small to have been able to ignite and become stars. Alternatively, they are very small stars that emit very little light. In either case, they would not appear as part of the visible matter in a galaxy. In this problem, we will build a vey simple model in which the density of dark matter, $\rho_{DM}(r)$ is zero inside the core of the galaxy, and has some unknown radial dependence outside the core. (a) Assume that the density of dark matter is given as

$$\rho_{DM}(r) = D_0\, f(r)$$

where D_0 is a constant and $f(r)$ is a function only of r (*e.g* $f(r) = 1/r$ or $f(r) = \sqrt{r}$). What form must $f(r)$ take to explain a flat rotation curve outside the core of the galaxy? In terms of the core radius, R_c and the core mass, M_c, determine the value of D_0 in our density expression. (b) Assuming that the dark matter extends out to a radius of about $10\,R_c$, what is the total mass of dark matter in the galaxy? Express this in terms of the core mass, M_c. (c) Assuming that this dark matter is made up of MACHOs that have masses of about 5% of a solar mass, how may of these MACHO do we expect in a typical galaxy? Experimental limits claim to have ruled out MACHOs with masses larger than 0.00000001 of a solar mass. Using this upper limit, how many of these MACHOS would we need? (d) Our solar system is about $2.5\,R_c$ from the center of our galaxy, or about $25,000\,ly$. What would we expect the number of MACHOs to be in a small sphere $100\,ly$ across centered on our Sun? Do this for both size limits from part c. For reference, the number of stars within $50\,ly$ of the Sun is about 2000, which is a fairly dense region of the galaxy.