

Name (print) \_\_\_\_\_ Section: \_\_\_\_\_

Instructor Signature \_\_\_\_\_

**Due Date:** Wednesday of Week 12 in Lecture.

## RUTHERFORD SCATTERING

### Introduction

This programming assignment is based on Problem 8.P65 of your textbook, Rutherford Scattering. We will study how Rutherford was able to deduce the size of the nucleus by looking at “scattering” of  $\alpha$ 's by the gold nucleus.

Ernest Rutherford carried out a set of experiments in the early part of the last century that showed that the nucleus of an atom contained all of the positive charge, and was quite small. A picture of the atom that was popular at the time was the so called “Plum-Pudding” model which viewed the atom as a blob (the pudding) in which the “electrons” were embedded like “plums”. The atomic sizes were roughly known as  $10^{-10} m$  and given that the atom was effectively neutral, there was no long extreme electric force effect expected. It was also known that  $\alpha$  particles (we now know that they are  ${}^4He$  nuclei) were positively charged, relatively compact objects. Rutherford shot a beam of  $10 \times 10^6 eV$ ,  $\alpha$  particles (10 MeV Kinetic Energy) at a thin gold foil, and was watching how the  $\alpha$ 's were deflected. They were expected to be scattered mostly at small angles in the forward direction, and nothing was expected in the backward direction. The striking result was that  $\alpha$ 's were observed bouncing backward off the gold atoms. Rutherford's conclusion was that the positive charge was all in a tiny nucleus in the middle of a mostly empty atom.

Before we can proceed, we need to review the Coulomb force law. For two point charges a distance  $r$  apart, the force is given as:

$$\vec{F}_{1on2} = \frac{1}{4\pi\epsilon_o} \frac{q_1q_2}{r^2} \hat{r}_{1to2}$$

where  $q_i$  are the charges in Coulombs,  $r$  is the distance between them in meters and  $\frac{1}{4\pi\epsilon_o} = 9 \times 10^9 Nm^2/C^2$ . In contrast, for a point particle (the  $\alpha$ ) that penetrates inside uniform spherical charge distribution of total radius  $R$ , the electric force *inside* the sphere can be shown to be:

$$\vec{F}_{1on2} = \frac{1}{4\pi\epsilon_o} \frac{q_1q_2}{R^3} \vec{r}_{1to2}$$

where  $\vec{r}$  is the vector from the center of the charge distribution to the  $\alpha$  particle.

### Assignment

- 8.P65. Write a program that shoots an  $\alpha$  particle of  $10 \times 10^6 eV$  of kinetic energy DIRECTLY at the center of a gold nucleus of radius  $R$ . Start out with  $R$  about  $10^{-12}$  meters and decrease it until you can get the  $\alpha$  to bounce back. What is the maximum size of the



Figure 1: The  $\alpha$  being shot directly at the center of the gold nucleus.

gold nucleus for bounce-back to occur? You should set up your program so that the  $\alpha$  starts some reasonable distance away from the center of the nucleus, and stops looping when it gets farther than the initial distance from the nucleus.

- 8.P66. Using your program from problem 8.P65, set the size of the nucleus to be about  $\frac{1}{2}$  of the maximum size you found. Now modify your program to shoot the  $\alpha$  particle at the gold nucleus, but off axis by some distance  $b$  (the impact parameter). Vary  $b$  from about  $10 \times R$  to 0 and observe what happens. Now modify your program to print out the angle at which the  $\alpha$  is moving when your loop stops, and then make a plot of  $\theta$  versus  $b$  from your data. (You can make the plot by hand). How small must  $b$  be for the  $\alpha$  to be deflected through an angle of  $60^\circ$ ? Of  $90^\circ$ ? Of  $120^\circ$ ? Of  $150^\circ$ ?

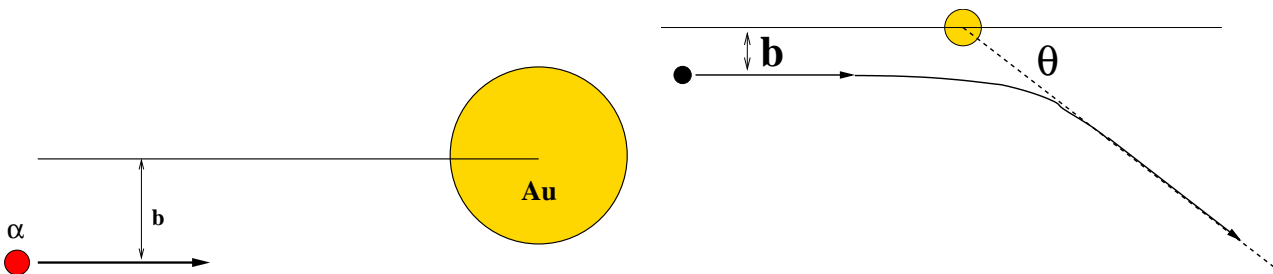


Figure 2: The  $\alpha$  being shot off axis at the gold nucleus.

- 8.P67. Modify your program from 8.P66 to include the recoil of the gold nucleus. Does this impact the results that you found?

On a separate sheet of paper, put your answers to all the questions asked in this write-up, and staple it, together with your plot, to these pages. Have a TA sign off on your work before you hand it in.

## Programing Hints

To be able to finish this program in a reasonable amount of time, it will be necessary for you to carry out a number of calculations to determine reasonable values for your program. Before we start, we need to make sure that we have the correct MKS values for a number of constants.

- What is the mass of an  $\alpha$  in kg?
- What is the mass of a gold nucleus in kg?
- An alpha has a charge of  $+2e$ , what is this in Coulombs?
- A gold nucleus has 79 protons in it, what is its charge in Coulombs?
- The  $\alpha$  has a kinetic energy of  $10 \times 10^6$  eV. What is its speed and momentum in MKS units? Do we need to worry about relativistic effects?
- Given two point charges as calculated above, and a total kinetic energy as given above, how close can the two charges get to each other?
- For our program, we know that the nucleus is measured in units of  $10^{-15}$  m, and we want to have a small but finite number of steps through this distance. What should we use for  $\Delta t$ ?

## Program Shell

```

from math import *
from vpython import *
#
#
# Because the force is different if the alpha is outside or inside the gold
# nucleus, we will use a function to compute the force. You need to code up
# both cases here for the force between part1 and part2
#
def force(part1,part2):
    if ( mag(part1.pos-part2.pos) >= part2.radius ) :
        force = 0
    else:
        force = 0
    return force
#
scene2=canvas(title='Rutherford Scattering',caption='Caption',center=vector(0,0,0))
#
# Mass of the Gold:      3.35e-25 kg
# Radius of the Gold:   5.0e-14
# Mass of the Alpha:    6.8e-27 kg
# Radius of the Alpha   1e-15 m
# Kinetic Energy of the Alpha  1.6e-12 Joules
#
# size is a scale factor that we use. Don't change
#

```

```

size=4.0e-13
#
# Initialzie parameters:
# Kinetic Energy of alpha in Joules
#
GoldRadius = 0
AlphaRadius = 0
AlphaEnergy = 0
ImpactParameter = 0
scene2.caption=('Impact parameter=',ImpactParameter)
#
# Electric Force Constant
#
k = 0
elechg = 0
#
# Initialize time to zero, and set a time step (seconds)
t = 0
dt = 1.0e-23
#
# Create the Alpha Particle
#
alpha = sphere(pos=vector(-0.95*size,ImpactParameter,0), radius=AlphaRadiue, color=color
gold = sphere(pos=vector(0,0,0), radius=GoldRadius,color=color.yellow,opacity=0.25)
#
# You need to fill in reasonable values
#
alpha.mass = 0
alpha.charge = 0*elechg
alpha.energy = AlphaEnergy
p_alpha = (2.0*alpha.mass*alpha.energy)**0.5 # Non-relativistic
#
alpha.momentum = vector(p_alpha,0,0)
alpha.velocity = vector(p_alpha/alpha.mass,0,0)
#
print('Alpha mass = ',alpha.mass)
print('Alpha momentum = ', alpha.momentum)
print('Alpha Velocity = ', alpha.velocity)
#
# Put in reasonable values
#
gold.momentum=vector(0,0,0)

```

```

gold.mass = GoldMass
gold.charge = 0 * elechg
#
# Commands to set up a graph.....
#
s='<b>Rutherford </b>'
graph(title=s, xtitle='Time', ytitle='Angular Momentum', xmin=0, xmax=6*size, ymax=15, y
#
gold.trail = curve(color=gold.color)
alpha.trail = curve(color=alpha.color)
#
runit = 1
#
while (runit==1):
#
    rate(100)
#
# Calculate the force exerted on the alpha by the gold. If
# the alpha is outside the gold, it is coulomb, inside it
# is linear.
#
    alpha.force = force(alpha,gold)
#
# Update Mometum and positions
#
    alpha.momentum = 0
    alpha.pos = 0
#
    gold.momentum = 0
    gold.pos = 0
#
    alpha.trail.append(pos=alpha.pos)
    gold.trail.append(pos=gold.pos)
#
    t = t+dt
#
# This is a stopping condition that if we get farther away than size,
# we stop, or if we are running forever.
#
    if(mag(alpha.pos)>size):
        runit=0
    if(t>10):    # if more than 10 seconds have passed, we have a problem.

```

```
        runit=0
#
# Add stuff below after we have stopped looping
#
theta = atan2(alpha.pos.y,alpha.pos.x)*57.3
scene2.caption=('Impact parameter=',b,' Scattering angle =',theta)
#
```