

Charge exchange, OPE production amplitudes of $Y^0\pi^+$ (Y is the isobar) via a_1 , a_2 , π_2 and $\hat{\rho}$.

$$\gamma p \rightarrow X^+ n \rightarrow Y^0 \pi^+ n (\rightarrow \pi^+ \pi^- \pi^+ n) \quad (1)$$

$$\frac{d^2\sigma}{dt dM_{\rho\pi} d\Omega_k} = \frac{1}{2} \sum_{\lambda_N \lambda_{N'} \lambda_Y} \frac{389.3 \mu\text{b GeV}^2}{64\pi m_N^2 E_\gamma^2} \frac{|\mathbf{k}_Y|}{2(2\pi)^3} |A(s, t, M_{Y\pi}, \Omega_k, \lambda_\gamma, \lambda_N, \lambda'_N, \lambda_Y)|^2 \quad (2)$$

Spin projections of the isobar, λ_Y and resonances, λ_X , $M_{Y\pi}$, (see below) are defined in the GJ frame *i.e.* resonance at rest, photon momentum along the z axis, (λ_γ =photon helicity). In contrast, nucleon will be parametrized by its helicity.

The amplitude A is written as a product of the production amplitude A_{OPE} (production as defined by Dennis) and the amplitude T for the resonance (assumed BW) to decay into $Y\pi$. (I think Dennis has that in his code, I'm giving it here to completeness),

$$A = \sum_{X, J^P, \lambda_X} A_{OPE}(s, t, \lambda_N, \lambda_\gamma, \lambda'_N, J^P, \lambda_X) T(M_{Y^0\pi^+}, \Omega_k, J^P, \lambda_X, \lambda_Y) \quad (3)$$

$$\begin{aligned} A_{OPE}(s, t, \lambda_N, \lambda_\gamma, \lambda'_N, J^P, \lambda_X) &= \sqrt{2} g_{\pi NN} \left(\frac{s}{s_0}\right)^{\alpha_\pi(t)} \frac{\sqrt{t'}}{(t - m_\pi^2)} \beta(t) \delta_{\lambda_N, -\lambda'_N} \\ &\times \sum_{L_\gamma} \sqrt{\frac{2L_\gamma + 1}{4\pi}} \langle J, \lambda_X | L_\gamma 0, 1 \lambda_\gamma \rangle \left(\frac{\omega}{m_X}\right)^{L_\gamma} g_{X\gamma\pi}^{L_\gamma}. \end{aligned} \quad (4)$$

Here $t' = |t - t_{min}|$, $\omega = \lambda(m_X, 0, t)$ is the breakup momentum of the resonance X with mass m_X decaying into photon and an off-shell pion with mass t , $g_{\pi NN}^2/(4\pi) \sim 14.4$, $s = 2E_\gamma m_n + m_n^2$, and $\alpha_\pi = 0.9 \text{GeV}^{-2}(t - m_\pi^2)$. The form factor $\beta(t)$ was fitted to the $\gamma p \rightarrow a_2^+ n$ data from Ref. [2], [3] and is given by

$$\beta(t) = (a_1 e^{b_1 t} + a_2 e^{b_2 t})^{1/2} \quad (5)$$

with $b_1 \sim 35 \text{GeV}^{-2}$, $b_2 \sim 3 \text{GeV}^{-2}$, $a_1 \sim 30$, $a_2 \sim 0.05$ for $s_0 = 10 \text{GeV}^2$.

$$\begin{aligned} T(M_{Y^0\pi^+}, \Omega_k, J^P, \lambda_X, \lambda_Y) &= \sum_{L_{Y\pi} M_{Y\pi}} \frac{m_X g_{XY^0\pi^+}^{L_{Y\pi}}}{2(M_{Y^0\pi^+} - m_x + i \frac{\Gamma_{X, J^P}}{2})} \left(\frac{k_{Y\pi}(m_X)}{m_x}\right)^{L_{Y\pi}} \\ &\times \langle s_Y \lambda_Y, L_{Y\pi} M_{Y\pi} | J_X \lambda_X \rangle Y_{L_{Y\pi}, M_{Y\pi}}(\Omega_k) \end{aligned} \quad (6)$$

The photocouplings $g_{X\gamma\pi}^{L_\gamma}$ are calculated from the radiative widths $\Gamma_{X\gamma\pi}$ using

$$\Gamma_{X\rightarrow\gamma\pi}(E) = m_X \sum_{L_\gamma, L'_\gamma} \frac{g_{X\gamma\pi}^{L_\gamma} g_{X\gamma\pi}^{L'_\gamma}}{32\pi^2} \left(\frac{q}{m_x}\right)^{L_\gamma+L'_\gamma+1} \times \left[\delta_{L_\gamma, L'_\gamma} \left(\delta_{L_\gamma, J} + \delta_{L_\gamma, J+1} \frac{J}{2J+1} + \delta_{L_\gamma, J-1} \frac{J+1}{2J+1} \right) + \left(\delta_{L_\gamma, J+1} \delta_{L'_\gamma, J-1} + \delta_{L'_\gamma, J+1} \delta_{L_\gamma, J-1} \right) \frac{\sqrt{J(J+1)}}{2J+1} \right], \quad (7)$$

with $q = \lambda(m_X, 0, m_\pi)$. Note that, if there is more than one partial amplitude, L_γ contributing, $\Gamma_{X\gamma\pi}$ determines only a product of different couplings. In such a case, to completely determine all $g_{X\gamma\pi}^{L_\gamma}$'s we assumed that the amplitude ratios are the same as for $X \rightarrow \rho\pi$.

The strong couplings are computed from

$$\Gamma_{X, J^P}^{L_{Y\pi}} = m_X \frac{(g_{XY\pi}^{L_{Y\pi}})^2}{32\pi^2} \left(\frac{k_{Y\pi}}{m_X}\right)^{2L_{Y\pi}+1}. \quad (8)$$

Here $k_{Y\pi} = \lambda(m_X, m_Y, m_\pi)$ is the resonance breakup momentum. Photocouplings and strong couplings to selected $Y\pi$ channels are summarized in Table 2. These are calculated from Eqs. 7, 8 using, where known, the PDG values * for $\Gamma_{X\gamma\pi}$ and $\Gamma_{X, J^P}^{L_{Y\pi}}$ and “reasonable guesses” otherwise, (all listed in Table. 1).

*See caption for Table.1

REFERENCES

- [1] A. Afanasev and A.P. Szczepaniak almost finished.
- [2] Y. Eisenberg *et al.*, Phys. Rev. Lett. **23**, 1322 (1969).
- [3] G.T. Condo *et al.*, Phys. Rev. **D48**, 3045 (1993).

TABLES

Resonance	J^{PC}	Mass[GeV]	Partial waves	$\Gamma_{XAB}^{LAB}/\Gamma$	Γ [MeV]	Γ^e [keV]
decay channel			$(L^{AB}, L_\gamma = L^{\rho\pi})$			
a_1	1^{++}	1.26			400	640
$\rho\pi$			S	0.99		
			D	0.01		
a_2	2^{++}	1.32			110	295
$\rho\pi$			D	0.70		
$\eta\pi$			D	0.30		
π_2	2^{-+}	1.67			258	300
$\rho\pi$			P	0.98*0.31		
			F	0.02*0.31		
$f_2\pi$			S	0.69		
$\hat{\rho}$	1^{-+}	1.6			400	170
$\rho\pi$			P	0.50		
$b_1\pi$			S	0.50		

TABLE I. Resonance parameters. $\rho\pi$ widths are taken from the PDG. Γ is the total hadronic width, $\Gamma = \sum_{AB,L_{AB}} \Gamma_{XAB}^{LAB}$ and Γ^e is the total radiative width to $\gamma\pi$. Here I have assumed that resonances can be saturated by the (two body) decay channels listed above. This is close to reality, however in principle one can recalculate the couplings in Table 2 for the “true” PDG values”. (This assumption was made to make the theoretical model of Ref. [1] simpler).

Resonance	Partial wave	$\frac{(g_{XAB}^{L_{AB}}(k_{AB}(m_X)))^2}{4\pi}$	$\frac{(g_{X\gamma\pi}^{L_{\gamma}})^2}{4\pi}$
decay channel			
a_1			
$\rho\pi$	S	26.28	1.03
	D	32.54	1.04×10^{-2}
a_2			
$\rho\pi$	D	442.4	2.37
$\eta\pi$	D	57.08	72.71
π_2			
$\rho\pi$	P	20.01	1.74
	F	20.15	4.01×10^{-2}
$f_2\pi$	S	13.72	
$\hat{\rho}$			
$\rho\pi$	P	24.59	1.51
$b_1\pi$	S	7.12	

TABLE II. Strong and electromagnetic couplings corresponding to (BW) widths given in Table 1. Strong couplings, $g_{XY^0\pi^+}^{LY\pi}$ to the charged state $Y^0\pi^+$ when Y^0 is part of an isovector should be reduced by $\sqrt{2}$.