Momentum Dependence of the Decay $\eta \to \pi^+\pi^-\pi^0$

Crystal Barrel Collaboration

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The π^0 momentum dependence of the decay $\eta \rightarrow \pi^+\pi^-\pi^0$ has been measured with the Crystal Barrel detector. The analysis is based on 3230 events. The results of this independent measurement are compared to new chiral perturbation theory calculations and previous measurements.

The calculation of the amplitude of the η decay into three pions has been a longstanding problem. Both the current algebra prediction of $\Gamma_{\eta\rightarrow\pi^+\pi^-\pi^0} = 66$ eV and the first order chiral perturbation theory $\Gamma = (160 \pm 50)$ eV [1] has failed to predict the experimental value of $\Gamma = (274 \pm 26)$ eV [2]. The decay of the η to the $I = 1$ final state of the three pion system occurs primarily due to the $d - u$ quark mass difference:

$$F(s_a, s_b, s_c) = -\frac{B_0(m_d - m_u)}{3\sqrt{3}F_0^2} f^{(2)}(s_a)(1 + \delta(s_a, s_b, s_c)) \tag{1}$$

$$f^{(2)}(s_a) = 1 + \frac{3(s_a - s_0)}{m_\eta^2 - m_\pi^2},$$

$$s_a = (p_\eta - p_{\pi^0})^2, s_b = (p_\eta - p_{\pi^+})^2, s_c = (p_\eta - p_{\pi^-})^2,$$

where m_u and m_d are the up and down current quark masses, F_0 and B_0 are QCD parameters, $s_0 = (s_a + s_b + s_c)/3$ and δ is a higher order correction term. The lowest order amplitude is defined by the above equation, setting $\delta = 0$. The next-to-leading order corrections have been calculated by Gasser and Leutwyler [1] amounting to about 50%. Recent developments increase the value of Ref. [1] by either recalculating the $d-u$ quark mass difference [3,4] or by including more corrections (namely final state interactions) in δ [5]. While the former affects the absolute value, the latter modifies also the momentum distribution of the pions, especially s_a of the neutral π^0. For a full discussion of the current theoretical situation see Ref. [5].

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Though two high statistics measurements of the Dalitz plot $\eta \to \pi^+\pi^-\pi^0$ already exist [6,7], their fitted parameters of the distribution differ by several standard deviations. This letter describes a recent and independent measurement of this distribution.

Using antiprotons from LEAR the η mesons were produced in the reaction

$$\overline{p}p \to \pi^0\pi^0\eta \tag{2}$$

with a branching ratio of $(6.7 \pm 1.2) \cdot 10^{-3}$ [8]. The decays $\eta \to \pi^+\pi^-\pi^0$ and $\pi^0 \to \gamma\gamma$ lead to a total of 6 photons and 2 tracks in the final state. They were detected by the Crystal Barrel detector. The near 4π solid angle coverage of the Crystal Barrel detector allows a full reconstruction of the η and π^0 decay products.

A 200 MeV/c antiproton beam stops in the liquid hydrogen target at the center of a solenoidal magnet. The target is surrounded by two multiwire proportional chambers and a cylindrical jet drift-chamber (JDC) with 23 layers. The JDC is surrounded by an electromagnetic calorimeter, consisting of 1380 CsI(Tl) crystals pointing towards the target center. The calorimeter covers the polar angles between 12° and 168° with full coverage in azimuth. A detailed description of the detector can be found elsewhere [9].

The analysis is based on 10 million events taken with a 2-prong trigger. The trigger required exactly two hits in both PWC layers and two hits in the outer layers 19 and 20 of the JDC. Further event selection is done in the offline analysis. The photon energy has to be at least 20 MeV. Electromagnetic and hadronic "split-offs", i.e. signals in the calorimeter which do not occur from the primary photon shower or cannot be correlated with a charged particle in the JDC, are rejected by geometric cuts and a 4-C energy momentum conservation requirement. The helix fit to the tracks is required to have $\chi^2/N_{dof} < 1.5$. Final event selection is performed by a 7-C kinematic fit requiring total energy-momentum conservation and fixing the mass of the $3\pi^0$'s in the decay $\pi^0 \to \gamma\gamma$. Accepted events have to exceed a probability of 10%.

Events from reaction (2) with the decay

$$\eta \to \pi^+\pi^-\pi^0 \tag{3}$$

are identified by histogramming the $\pi^+\pi^-\pi^0$ invariant mass and fitting the η signal with a Gaussian. The combinatorial background is described by a Legendre polynomial of degree 4. The fit gives $N_\eta = 3230 \pm 87$, $m_\eta = 546.7 \pm 0.2$ MeV/c^2 and $\sigma_m = 6.2 \pm 0.2$ MeV/c^2 with $\chi^2/N_{dof} = 42.9/34$. The result is shown in Fig. 1.

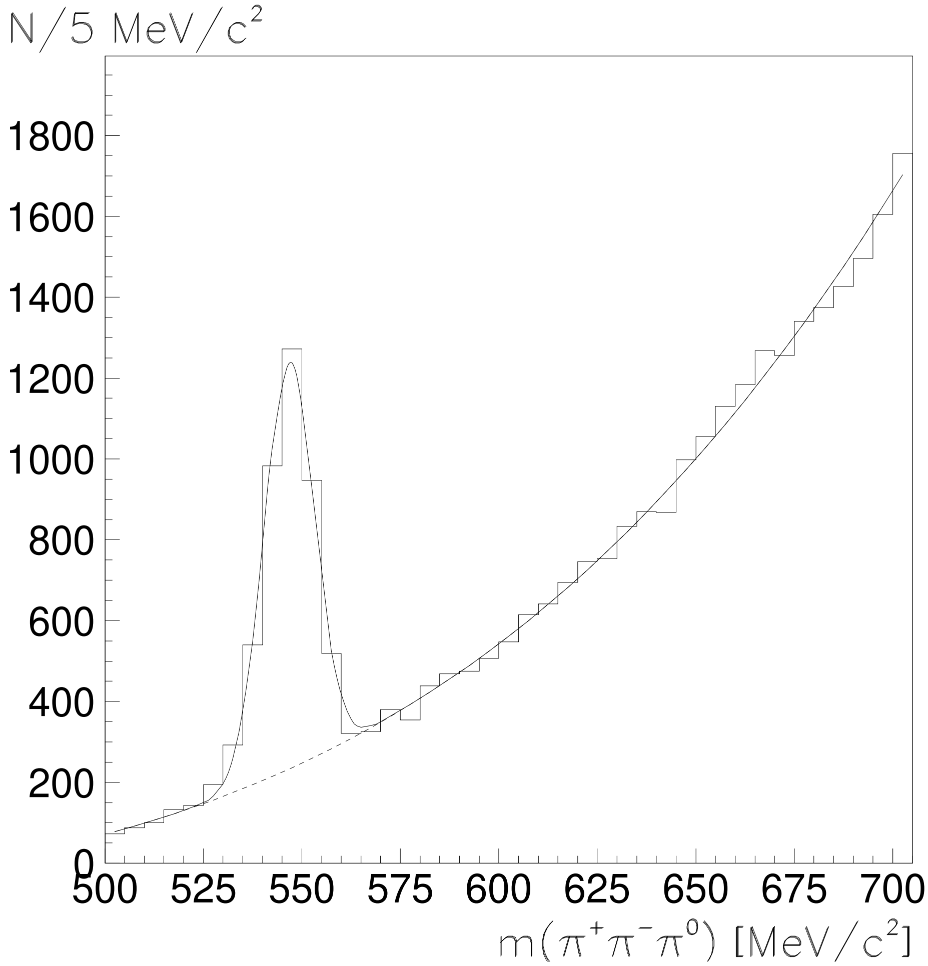

Fig. 1. Invariant $\pi^+\pi^-\pi^0$ mass of the reaction $\overline{p}p \to \pi^+\pi^-3\pi^0$. The line shows the Gaussian fit with the background Legendre polynomial of degree 4.

To parametrize the π^0 momentum dependence we use the usual Dalitz plot variables x and y:

$$
\begin{aligned}
x &= \sqrt{3}\frac{T_+ - T_-}{Q} = \frac{\sqrt{3}}{2m_\eta Q}(s_c - s_b), \\
y &= \frac{3T_0}{Q} - 1 = \frac{3}{2m_\eta Q}\{(m_\eta - m_{\pi^0})^2 - s_a\} - 1, \\
Q &= T_+ + T_- + T_0 = m_\eta - 2m_{\pi^+} - m_{\pi^0},
\end{aligned}
$$

where T_+, T_- and T_0 are the kinetic energies of the π^+, π^- and π^0, respectiveley. We then determine the distribution of the η decay matrix element $M^2(y)$ as follows: A $\pi^+\pi^-\pi^0$ invariant mass histogram is created for each interval in y, (-1.0,-0.9), (-0.9,-0.8) ... (0.8,0.9), resulting in 19 histograms. To calculate

y we use the measured kinetic energies in $Q = T_+ + T_- + T_0$. In each of these histograms the number of reconstructed η-mesons are determined by a fit to a Gaussian and polynomial of degree 4. In these fits we constrain the η mass to $m_\eta = 546.7 \pm 0.2$ MeV$/c^2$, which is obtained from the fit to the full spectra. This compensates for small deviations in the absolute calibration. The reconstruction efficiency is calculated by a Monte Carlo simulation of reaction (2) including a detector simulation using GEANT [10] and by applying the same analysis cuts as for real data. The number of reconstructed η mesons is then corrected by the efficiency of 3.5%. The resulting distribution of the matrix element $M^2(y)$ is shown in Fig. 2. In this figure we assumed $M^2(y)$ to be independent of x.

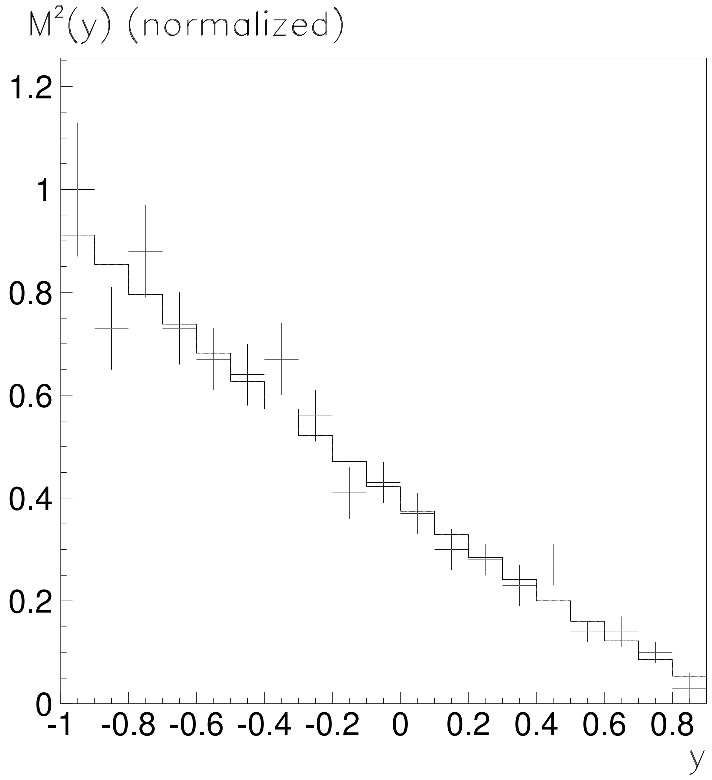

Fig. 2. Phase space normalized projection of the $\eta \to \pi^+\pi^-\pi^0$ Dalitz plot onto the y-axis. Shown are the data points and a fit to our data using the function $N(1 + ay + by^2)$ (solid line).

The largest source of systematic error is due to the background under the η signals in the $\pi^+\pi^-\pi^0$ invariant mass spectra, for example combinatorial background from $\overline{p}p \to \pi^0\pi^0\omega, \omega \to \pi^+\pi^-\pi^0$. Uncertainties in the background determination are already included in the error given by the fit. The stability of the fit procedure was checked by varying the order of the background polynomial and by doubling the bin size in the fits and comparing the numbers to the result from the individual fits. No significant differences were found.

Errors in the absolute Monte Carlo efficiency cancel, because the spectrum is normalized to the first bin, and thus only relative numbers are used.

To compare with theoretical and experimental results we fit the distribution to the function:

$$M^2(x, y) = N(1 + ay + by^2 + cx^2). \tag{4}$$

In our data the dependence on x was not directly measured. We integrate over x in the various y bins. Therefore the fit of parameter c is not very meaningful. It was fixed to values obtained from theory and other experiments. The values of a and b are essentially independent of c within the observed range of c.

Experiment	a	b	c	χ^2/N_{dof}
This work (c fixed)	-1.21 ± 0.07	0.21 ± 0.11	0.046 [7]	14.1/16
This work (c fixed)	-1.22 ± 0.07	0.22 ± 0.11	0.06 [6]	14.1/16
This work (c fixed)	-1.23 ± 0.07	0.23 ± 0.12	0.10 [5]	14.1/16
Layter et al.[7]	-1.08 ± 0.014	0.034±0.027	0.046±0.031	17.9/18
Gormley et al.[6]	-1.17 ±0.02	0.21 ± 0.03	0.06 ± 0.04	15.3/18
Theory	a	b	c	χ^2/N_{dof}
Gasser, Leutwyler [1]	-1.33	0.42	0.08	16.8/18
Kambor et al. (sol. a) [5]	-1.16	0.24	0.09	17.6/18
Kambor et al. (sol. b) [5]	-1.16	0.26	0.10	18.7/18

Table 1
Parameters for the distribution $M^2(x, y) = N(1 + ay + by^2 + cx^2)$. The χ^2/N_{dof} value is given for a fit to our data of the specified functions. For the fits to the other experiments and the theory no errors in a, b and c are taken into account.

Table 1 shows the result of the fits to our data together with the values obtained from the experiments of Layter et al. [7] and Gormley et al. [6] and the chiral perturbation theory values from Gasser and Leutwyler [1] and Kambor et al. [5]. All parametrizations are compatible with our data, though the fitted values of a and b of the Crystal Barrel data are closer to the ones from Kambor et al. [5] than those from Gasser and Leutwyler [1]. Our parameter a and b differ by 2 standard deviations from the value of Layter et al. [7].

Our data were also fitted assuming $c = 0$. Current algebra predicts no dependence on x, i. e. $c = 0$. The data is tested against the parametrization of two experiments and two theoretical predictions (table 2). In the second fit the parameter b was fixed to 0 to allow a better comparison to the result of Layter et al. [7].

Experiment	a	b	χ^2/N_{dof}	r
This work [1]	-1.19 $\pm$ 0.07	0.19 $\pm$ 0.11	14.1/16	1.44 $\pm$ 0.04
This work (b fixed)	-1.10 $\pm$ 0.04	0	17.2/17	1.43 $\pm$ 0.01
Layter et al.[7]	-1.07$\pm$0.013	0	17.7/18	1.44 $\pm$ 0.01
Gormley et al.[6]	-1.18$\pm$0.02	0.20$\pm$0.03	14.3/18	1.45 $\pm$ 0.01
Theory	a	b		
Current algebra [1]	-1.01	0.26	43.9/18	1.51
Chiral pert. theory [1]	-1.30	0.38	16.8/18	1.43$\pm$0.03

Table 2
Parameters for the distribution $M^2(y) = N(1 + ay + by^2)$. For the fits to the other experiments and the theory no errors in a and b are taken into account.

Our data is in good agreement with the other two experimental results and differs by less than 2 standard deviations from the chiral perturbation theory expectation. However the current algebra prediction, which is equivalent to the first order term of chiral perturbation theory is rejected by the data as the χ^2/N_{dof} value of 43.9/18 shows.

Following a procedure given in Ref. [1] one can calculate the value

$$r = \frac{\mathrm{BR}(\eta \to 3\pi^0)}{\mathrm{BR}(\eta \to \pi^+\pi^-\pi^0)}$$

from the parametrization of the matrix element $M^2(y) = N(1+ay+by^2)$. This method uses the assumption of an $I = 1$ decay to calculate the amplitude in $3\pi^0$ from the parameters a and b. The results are shown in the last column of table 2. Note that for the experiments this measurement of r is indirect, i. e. without measuring $\mathrm{BR}(\eta \to 3\pi^0)$ and contains only the error in a and b. No other systematic effects of this procedure are taken into account. This calculation is an additional check of the correctness of the parameters a and b. If the parametrization or the $I = 1$ assumption is wrong, one would not get the correct value of r. All experiments are compatible with the chiral perturbation prediction of r and the result of the direct measurements from the particle data group average of $r = 1.34 \pm 0.10$ and the global fit of $r = 1.39 \pm 0.04$ [2]. This value was measured by Crystal Barrel to be $r = 1.44 \pm 0.13$ [11]. Note that Kambor et al. [5] calculates $r = 1.40 \pm 0.03$.

In conclusion, we have measured the momentum distribution of the π^0 in the charged $\eta \to \pi^+\pi^-\pi^0$ decay. With fixed $c = 0.06$ we fit $a = -1.22 \pm 0.07$ and

[1] These values supersede an older Crystal Barrel measurement of $a = -0.94 \pm 0.15$ and $b = 0.11 \pm 0.27$ with $c = 0$, which was based on 1077 events [11].

$b = 0.22 \pm 0.11$. These values favor the measurement of Gormley et al. [6] over Layter et al. [7]. The current algebra prediction is rejected over the chiral perturbation theory results. The fitted values of a and b are closer to the improved chiral perturbation theory values calculated by Kambor et al. [5] than to the one loop result of Gasser and Leutwyler [1].

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