Technical Report of the
\( p\bar{p} \rightarrow \omega\omega, \omega \rightarrow \pi^0\gamma \) - Analysis
at Rest in \( LH_2 \)

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1 Data and Preselection

For the analysis of the reaction $p\bar{p} \rightarrow \omega\omega, \omega \rightarrow \pi^0\gamma$ in liquid hydrogen at rest all - neutral data from the run - periods June, July and November 1990 have been taken. Preselection - cuts rejected events, which could not satisfy simplest requirements for this reaction:

- no residual charged tracks in the JDC
- exactly 6 PEDs ($E_{PED} > 20\, MeV$)
- no PEDs in crystal type #13
- no split - offs (identified by smart)
- energy- an momentum window of $|\Delta E| = c |\Delta p| \leq 150\, MeV$

The data - reduction is shown in table 1.

<table>
<thead>
<tr>
<th>run - period</th>
<th># phys. events</th>
<th># no JDC tracks</th>
<th># 6 PED - events</th>
<th># no crystal #13</th>
<th># without split - offs</th>
<th># $E/\bar{p}$ - window</th>
</tr>
</thead>
<tbody>
<tr>
<td>June/July 1990</td>
<td>5332785</td>
<td>4788480</td>
<td>1048240</td>
<td>887554</td>
<td>831958</td>
<td>628838</td>
</tr>
<tr>
<td>Nov. 1990</td>
<td>4287832</td>
<td>3888401</td>
<td>842795</td>
<td>726064</td>
<td>683812</td>
<td>553305</td>
</tr>
</tbody>
</table>

Table 1: Data - reduction by preselection

2 Kinematic Fitting and Further Selection

The remaining data were subjected to a kinematic fit with CBKFIT to following hypotheses:

1. hypothesis 6γ (phase - space)
2. hypothesis $p\bar{p} \rightarrow \omega\omega, \omega \rightarrow \pi^0\gamma$
3. hypothesis $\pi^0\pi^0\gamma\gamma$
background:

4. hypothesis $\pi^0\pi^0\pi^0$
5. hypothesis $\pi^0\pi^0\eta$
6. hypothesis $\pi^0\eta\eta$
7. hypothesis $\eta\eta\eta$
8. hypothesis $\pi^0\pi^0\eta'$
9. hypothesis $\pi^0\eta\eta'$
10. $\pi^0\pi^0\omega$ mit $\omega \rightarrow \gamma\gamma$ (one $\gamma$ lost)

All mesons decay into photons. The error - factors were defined by normalized $\sqrt{E}$- and angular pulls of phase - space. Table 2 shows energy - resolution, error - factors for azimuthal - angles $\Phi$ and polar - angles $\Theta$ and their standard - deviations. The distributions of pulls and confidence - levels (cl) are shown in figures 1 and 2. All pulls except that of the energy are Gaussian - distributed with standard - deviations of $\sim 1$.

With the fit - results further selection - criteria were applied:

- Energy- and momentum conservation is guaranted by demanding a cl of 1% for the phase - space fit.
- The cl of hypothesis $\pi^0\pi^0\gamma\gamma$ must be higher than 10% and the cl's of all background - hypotheses. The $\omega$ - mesons were reconstructed by their decay pions and - photons. The invariant $\pi^0\gamma$ masses show a clear omega - signal (figure 3). By fitting the peak with a Voigt - profile one found an experimental width of $(14.76 \pm 0.20) MeV$.

<table>
<thead>
<tr>
<th>data</th>
<th>$\Delta \Phi$</th>
<th>$\sigma_{P,ki}$</th>
<th>$\Delta \Theta$</th>
<th>$\sigma_\Theta$</th>
<th>$\Delta E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>June/July 90</td>
<td>1.06</td>
<td>1.017</td>
<td>1.06</td>
<td>1.035</td>
<td>2.8 %</td>
</tr>
<tr>
<td>November 90</td>
<td>1.02</td>
<td>0.995</td>
<td>1.10</td>
<td>0.992</td>
<td>2.2 %</td>
</tr>
<tr>
<td>MC</td>
<td>1.02</td>
<td>1.046</td>
<td>1.10</td>
<td>1.051</td>
<td>2.0 %</td>
</tr>
</tbody>
</table>

Table 2: Multiplicators for angular - and energy - errors and the resulting pull - widths for different run - periods and Monte - Carlo events.
From the two possible $\pi^0\gamma$ combinations that one was used, where the sum of squared deviations from the $\pi^0\gamma$ masses to the $\omega$ - mass is minimum:

$$\left(m_{\pi^0\gamma}^1 - m_{\omega}\right)^2 + \left(m_{\pi^0\gamma}^2 - m_{\omega}\right)^2 \leq \text{Min}$$

The very rare events, where both combinations lie in a circle of 40 $MeV$ around the $\omega$ - mass, were rejected.

- In case of convergence of background hypotheses a cut demonstrated in figure 4 was applied. The cl of background hypotheses (in the example $\pi^0\pi^0\pi^0$) were plotted versus the cl of hypotheses 2. As selection - criteria a straight line were used. Slope and intercept have been adapted for each background - channel by means of MC studies.

- The energies of the pions and photons from the omega - decay have to lie in a window defined by kinematics. After this cut we receive the invariant $\pi^0\gamma$ - mass spectrum shown in figure 5.

- Having demanded a cl for hypothesis $p\pi\rightarrow\omega\omega$ $\omega \rightarrow \pi^0\gamma$ of 1% we only lost 5% of good events, while background was reduced considerably.

- At last only events were used, where the invariant $\pi^0\gamma$ - masses lie in a circle of 40 $MeV$ around the omega - mass.

<table>
<thead>
<tr>
<th>run - period</th>
<th># after</th>
<th># 4C - fit</th>
<th># $(\pi\pi\gamma\gamma)$ -</th>
<th>background:</th>
<th># $\omega$-</th>
<th>fraction of</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>presel.</td>
<td>converges</td>
<td>cl $\geq 10%$</td>
<td>cl $\leq ...$</td>
<td>events</td>
<td>total #</td>
</tr>
<tr>
<td>November 90</td>
<td>553305</td>
<td>397574</td>
<td>208300</td>
<td>26319</td>
<td>4984</td>
<td>$11.6 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>Juni/Juli 90</td>
<td>628838</td>
<td>483669</td>
<td>244255</td>
<td>35714</td>
<td>6596</td>
<td>$12.4 \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 3: Further reduction of preselected events.

The reduction of data by this selection - process can be taken from table 3.

### 3 Monte - Carlo Studies

In order to determine the efficiency of the selection - process and feed - through for background events, some channels with 7,6 and 5 photons in the
final state have been simulated by CBGEANT. The decay - products were
simulated phase - space distributed. The data - reduction of the generated
channels is shown in table 4. The detection - efficiency of reaction \( p\bar{p} \rightarrow \\

<table>
<thead>
<tr>
<th>channel</th>
<th># generated events</th>
<th># after presel.</th>
<th># (\pi\pi\gamma) - cl \geq 10%</th>
<th>background: cl \leq \ldots</th>
<th># after last cut</th>
<th>detection probab.</th>
</tr>
</thead>
<tbody>
<tr>
<td>\omega</td>
<td>100000</td>
<td>43918</td>
<td>22667</td>
<td>21100</td>
<td>18477</td>
<td>18.48%</td>
</tr>
<tr>
<td>\pi\pi\omega</td>
<td>120000</td>
<td>16560</td>
<td>4577</td>
<td>2903</td>
<td>207</td>
<td>0.17%</td>
</tr>
<tr>
<td>\pi\eta\omega</td>
<td>80000</td>
<td>9770</td>
<td>539</td>
<td>381</td>
<td>18</td>
<td>2.25 \cdot 10^{-4}</td>
</tr>
<tr>
<td>\pi\pi\pi</td>
<td>100000</td>
<td>44089</td>
<td>20761</td>
<td>1331</td>
<td>1</td>
<td>10^{-5}</td>
</tr>
<tr>
<td>\pi\eta \eta</td>
<td>30000</td>
<td>13325</td>
<td>3168</td>
<td>248</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>\eta \eta</td>
<td>20000</td>
<td>9264</td>
<td>323</td>
<td>54</td>
<td>5</td>
<td>2.25 \cdot 10^{-4}</td>
</tr>
<tr>
<td>\pi\pi\eta</td>
<td>5000</td>
<td>2486</td>
<td>23</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>\pi\eta \eta</td>
<td>20000</td>
<td>8541</td>
<td>2041</td>
<td>157</td>
<td>1</td>
<td>5 \cdot 10^{-5}</td>
</tr>
<tr>
<td>\pi\eta</td>
<td>5000</td>
<td>2270</td>
<td>101</td>
<td>19</td>
<td>1</td>
<td>2 \cdot 10^{-4}</td>
</tr>
<tr>
<td>\omega \pi</td>
<td>5000</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>\omega \eta</td>
<td>5000</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>\omega \eta</td>
<td>5000</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4: Reduction of monte - carlo data by the selection. The last column
contains the probability for the identification of an event as reaction \( p\bar{p} \rightarrow \\
\omega\omega, \omega \rightarrow \pi^0\gamma \).

\( \omega\omega, \omega \rightarrow \pi^0\gamma \) is 18.5 \pm 1.0\%. All background channels, even the strong
three pion channel, could be suppressed quite well. The only remarkable rest
origines from the reaction \( \pi^0\pi^0\omega \), where a low - energy photon has not been
reconstructed. Five - photon channels do not contribute at all.

4 The Branching - Ratio of the Reaction
\( p\bar{p} \rightarrow \omega\omega \)

Because of the enrichment of neutral events during the data - acquisition, first
the corresponding total number of annihilations \( N_G \) had to be determined.
The taken inverse enrichment - factor \( \epsilon_0 \) was (3.9\pm0.3)\%[2]. Furthermore \( N_G \)
is reduced by two correction - factors, because not all annihilations happened at rest and in the target:

\[
\begin{align*}
\epsilon_{Rah} &= (94.3 \pm 1.1)\% \\
\epsilon_{Target} &= (96.1 \pm 0.7)\%
\end{align*}
\]

\(N_0\) being the number of zero - prong events, \(N_G\) is calculated by

\[
N_G = \frac{N_0 \cdot \epsilon_{Rah} \cdot \epsilon_{Target}}{\epsilon_0}
\]

\[
= (123.7 \pm 9.7) \cdot 10^6 \quad \text{for June/July 1990}
\]

\[
= (99.5 \pm 7.8) \cdot 10^6 \quad \text{for November 1990}
\]

With the known background branching ratios (table 5) and feed - through probabilities received by monte - carlo (table 3), the contributing background in the final sample could be calculated. The error of the efficiencies was

<table>
<thead>
<tr>
<th>channel</th>
<th>branching - ratio</th>
<th>fraction in 5, 6 or 7 photons</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi\pi\omega)</td>
<td>((2.00 \pm 0.21) \cdot 10^{-2})</td>
<td>((0.164 \pm 0.027) \cdot 10^{-2})</td>
</tr>
<tr>
<td>(\pi\eta\omega)</td>
<td>((0.68 \pm 0.01 \pm 0.05) \cdot 10^{-2})</td>
<td>((0.022 \pm 0.002) \cdot 10^{-2})</td>
</tr>
<tr>
<td>(\pi\pi\pi)</td>
<td>((0.62 \pm 0.10) \cdot 10^{-2})</td>
<td>((0.60 \pm 0.16) \cdot 10^{-2})</td>
</tr>
<tr>
<td>(\pi\pi\eta)</td>
<td>(0.66 \cdot 10^{-2})</td>
<td>(0.25 \cdot 10^{-2})</td>
</tr>
<tr>
<td>(\pi\pi\eta')</td>
<td>((0.20 \pm 0.06) \cdot 10^{-2})</td>
<td>((0.030 \pm 0.010) \cdot 10^{-2})</td>
</tr>
<tr>
<td>(\eta\eta\eta)</td>
<td>(\pi\pi\eta')</td>
<td>(\pi\eta\pi)</td>
</tr>
<tr>
<td>(\omega\pi)</td>
<td>((5.73 \pm 0.47) \cdot 10^{-4})</td>
<td>((0.481 \pm 0.0286) \cdot 10^{-4})</td>
</tr>
<tr>
<td>(\omega\eta)</td>
<td>((1.51 \pm 0.12) \cdot 10^{-2})</td>
<td>((4.99 \pm 0.36) \cdot 10^{-4})</td>
</tr>
<tr>
<td>(\omega\eta')</td>
<td>((0.78 \pm 0.08) \cdot 10^{-2})</td>
<td>((0.144 \pm 0.019) \cdot 10^{-4})</td>
</tr>
</tbody>
</table>

Table 5: Possible background - channels and, if known, their branching - ratios into 5,6 or 7 photon final states

estimated by a statistic error and systematic part of 5.2\% [1].
\[
\begin{align*}
\epsilon_{\omega\omega} & = (18.5 \pm 0.1_{\text{stat.}} \pm 1.0_{\text{syst.}}) \% \\
\epsilon_{\pi\pi\omega} & = (17.3 \pm 1.2_{\text{stat.}} \pm 0.9_{\text{syst.}}) \cdot 10^{-4} \\
\epsilon_{\pi\pi\pi} & = (1 \pm 1_{\text{stat.}}) \cdot 10^{-5} \\
\epsilon_{\pi\eta\eta} & = (2.3 \pm 1.0_{\text{stat.}} \pm 0.1_{\text{syst.}}) \cdot 10^{-4} \\
\epsilon_{\pi\eta\omega} & = (2.3 \pm 0.5_{\text{stat.}} \pm 0.1_{\text{syst.}}) \cdot 10^{-4}
\end{align*}
\]

Thus have following remaining background contribution:

<table>
<thead>
<tr>
<th>June/July 1990</th>
<th>November 1990</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_{\pi\pi\omega})</td>
<td>(352 \pm 59)</td>
</tr>
<tr>
<td>(N_{\pi\pi\pi})</td>
<td>(11 \pm 11)</td>
</tr>
<tr>
<td>(N_{\pi\eta\eta})</td>
<td>(9 \pm 4)</td>
</tr>
<tr>
<td>(N_{\pi\eta\omega})</td>
<td>(6 \pm 1)</td>
</tr>
</tbody>
</table>

Subtracted from the surviving number of events one got \(N_{\omega\omega} = (6218 \pm 154)\) good events for June/July 1990 and \((4680 \pm 125)\) for November, which can be regarded as “real” \(\omega\omega\) - events. With the efficiency \(\epsilon_{\omega\omega}\) and the decay - probabilities of the mesons the branching - ratio for \(p\bar{p} \rightarrow \omega\omega\) is

\[
BR(p\bar{p} \rightarrow \omega\omega) = \frac{N_{\omega\omega}}{N_G \cdot \epsilon_{\omega\omega} \cdot BR^2(\omega \rightarrow \pi^0\gamma) \cdot BR^2(\pi^0 \rightarrow \gamma\gamma)} = \frac{(3.86 \pm 0.55)\%}{(3.62 \pm 0.52)\%}
\]

for June/July 1990 and November 1990, respectively.

This results an average - value of \((3.74 \pm 0.54)\%\), consistent with the published value of \((3.32 \pm 0.34)\%\) [3]. Errors are strongly dominated by the systematic errors of \(BR(\omega \rightarrow \pi^0\gamma) = (8.5 \pm 0.5)\%\) and \(\epsilon_0 = (3.9 \pm 0.3)\%\), which are the same for both runs. Statistical errors are negligible (< 1%), so that the total errors of both run - periods must be averaged arithmetically.

5 Determination of Angular - Momenta

For determination of initial angular - momenta contribution of protonium the event - topology was fitted by a log - likelihood fit. The log - likelihood
was defined by

\[ l = - \log N! \sum_{i=1}^{N} \log w_i + N \log \left( \Phi \frac{N}{N_c} \right) + \frac{1}{2} \left( \frac{\Phi}{N_c} - 1 \right)^2, \tag{1} \]

where \( N \) and \( N_c \) are the number of data- and monte-carlo events and \( w_i \) the weights of the data-events. \( \Phi \) is the sum over all monte-carlo weights. Thus defined log-likelihoods we need not care about efficiencies. The weights are products of a dynamical part and phase space. The former was calculated in the helicity-formalism from the angular-distribution in the reaction. The resulting scattering-amplitude \( A_{\lambda_1 \lambda_2}^{JM} \) was squared and summed incoherent over the photon helicities and initial angular-momenta \( J \) of the assumed \( ^1S_0, ^3P_0, ^3P_1, \) and \( ^3P_2 \) - states:

\[ A_{\lambda_1 \lambda_2}^{JM} = \sum_{\lambda_1 \lambda_2} \sqrt{2J+1} (2s+1) D_{JM}(\Omega)^* D_{\lambda_1 \lambda_2}(\Omega_1)^* D_{\lambda_2 \lambda_1}(\Omega_2)^* F_{\lambda_1 \lambda_2}^{J} f_{\lambda_1 0} f_{\lambda_2 0} \tag{2} \]

The angular dependent parts in the amplitude are given by the standard \( D \)-functions. \( \Omega \) means the azimuthal and polar production-angle \( \Phi \) and \( \Theta \) of one of the \( \omega_s \), \( \Omega_i \) means the decay-angles of the \( i \)-th \( \omega \) in its rest-system. An initial spin-density of \( \frac{1}{2J+1} \) was assumed, so that the distributions of the production-angles should be flat because of the unitarity of the \( D \)-functions. In figure 6 the cosine of production polar-angle \( \Theta \), the cosines of the decay polar-angles \( \vartheta_i \) and the sum and the difference of the decay azimuthal-angles \( \phi_{\pm} = \phi_i \pm \phi_2 \) are plotted. The lower pictures show the distributions corrected (one dimensional) by efficiencies. This is sufficient for recognizing the main structures. They are not consistent with a pure \( S \)-wave state, where following intensity-distributions are expected:

\[ \mathcal{I}(\vartheta_{1/2}) = \int \mathcal{I} \, d\vartheta_{1/2} \, d\Phi_+ \sim 1 + \cos^2 \vartheta_{1/2} \tag{3} \]

\[ \mathcal{I}(\Phi_+) = \int \mathcal{I} \, d\vartheta_1 \, d\vartheta_2 \sim 3 + 2 \sin^2 \Phi_+ \tag{4} \]

(\( \Phi_- \) - distribution should be flat)

More details result from the fit to the data. For minimizing \( l \) the MINUIT-package was used. Fit-parameters were the helicity-amplitudes \( F_{\lambda_1 \lambda_2}^{J} \) from the omega production, developed in partial-wave amplitudes \( a_{l}^{J} \).
according to the relation

\[
F^{J}_{\lambda_1\lambda_2} = \sum_{l_0} \sqrt{\frac{2l + 1}{2J + 1}} a_{l_0}(J\lambda_1 | l_0 s \lambda)(s \lambda | s_1 \lambda_1 s_2 - \lambda_2).
\]

(5)

The fit converged with the amplitudes and their statistical errors of table 6. With this amplitudes the fraction of angular - momentum states in protonium

<table>
<thead>
<tr>
<th>(a^0_{22} )</th>
<th>( a^0_{23} )</th>
<th>( a^0_{24} )</th>
<th>( a^0_{25} )</th>
<th>( a^0_{26} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.737 ± 0.034</td>
<td>1.221 ± 0.050</td>
<td>0.153 ± 0.031</td>
<td>0.071 ± 0.092</td>
<td>0.099 ± 0.088</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(a^2_{22} )</th>
<th>( a^2_{23} )</th>
<th>( a^2_{24} )</th>
<th>( a^2_{25} )</th>
<th>( a^2_{26} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005 ± 0.070</td>
<td>0.151 ± 0.075</td>
<td>0.066 ± 0.043</td>
<td>0.000 ± 0.076</td>
<td>0.007 ± 0.105</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(a^4_{22} )</th>
<th>( a^4_{23} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.145 ± 0.048</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Absolute values of the partial - wave amplitudes \(a^J_{2a}\)

could be determined:

\[
N(^1S_0) = (70.3 ± 3.1) \%
N(^3P_0) = (26.7 ± 2.1) \%
N(^3P_1) = (0.7 ± 1.0) \%
N(^3P_2) = (2.4 ± 1.3) \%
\]

Because of not having taken care of remaining background in this fit, these values may differ by about 5% due to the remaining \(\pi^0\pi^0\omega\) events, under the assumption, that phase - space simulation for this background was sufficient. Demanding a c l of 10%, 20% or 30% for hypotheses \(p\bar{p} \rightarrow \omega\omega, \omega \rightarrow \pi^0\gamma\) reduces \(\pi^0\pi^0\omega\) - background by 30 to 50%, while fitted parameter - values do not change significantly.

Nevertheless, the contribution of \(P\) - wave is not neglegable in \(p\bar{p}\) - annihilation to \(\omega\omega\).

References

[1] M. Merkel, Proton - Antiproton Vernichtung in \(\pi^0X\), \(\eta X\) und \(\omega X\) mit \(X = \pi^0, \eta \) und \(\eta'\), Dissertation, Mainz (1993)
[2] The Crystal - Barrel Collaboration, \( p\bar{p} \) - Annihilation at Rest into \( \pi\pi\omega \), Phys. Let. B 311, 362-370 (1993)

Figure 1: Distributions of pulls (above June/July 1990, below November 1990 data)
Figure 2: Distributions of confidence - levels
Figure 3: Invariant $\pi^0\gamma$ - masses (four entries per event) with a confidence level for hypothesis $\pi^0\pi^0\gamma > 10\%$
Figure 4: the cut in the confidence - level of background hypotheses \( \pi^0 \pi^0 \pi^0 \) versus hypothesis \( \mu \mu \rightarrow \omega \omega, \omega \rightarrow \pi^0 \gamma \).
Figure 5: Invariant $\pi^0\gamma$ - masses
Figure 6: Angular distributions of omegas and their decay photons. The lower picture shows the distributions corrected on efficiencies.